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DISCUSSIONS

APPLICATIONS FOR ADMISSION AND TRANSFER

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Discussion	May, Sept., Oct., Nov., 1933	Closed
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MEMBERSHIP

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

ANALYSIS OF SHEET-PILE BULKHEADS

By Paul Baumann,[1] M. Am. Soc. C. E.

Synopsis

The introduction of the deep-arch, steel, sheet-pile section in 1912 has gradually led to the construction of bulkheads of unprecedented height. During this development the designer has been confronted by serious difficulties because of the uncertainties as to the effective passive resistance of the ground and the interlock efficiency of the piles. Only since the end of the World War have large-scale experiments on these important question been made in the United States and Europe, the outstanding ones on passive resistance of soil being those by Professor O. Franzius in the hydraulic laboratory of the Institute of Technology, Hanover, Germany.

The exploration of the phenomenon of passive resistance of soil still being in its infancy, the present design of bulkheads is based on the convenient assumption that passive resistance of soil may be represented by static forces that vary linearly with the depth, irrespective of the magnitude of movement of the wall. The elastic behavior of the soil is thereby ignored.

The object of this paper is: (1) To outline and to interpret the result of full-sized tests on a steel sheet-pile bulkhead, particularly in regard to the effective, passive resistance of sand and the interlock efficiency of deep-arch sheet-piling; (2) to offer suggestions pertaining to a possible stress distribution in the passive prism between the wall and the surface of rupture; and (3) to present a new theory of the stability of bulkheads, which takes into consideration the elasticity of both wall and soil and which treats the problem as statically indeterminate.

Note.—Discussion on this paper will be closed in August, 1934, *Proceedings*.

[1] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs. Consolidated, Los Angeles, Calif.

Introduction

During the construction, in 1929-1930, of the Outer Harbor of the City of Long Beach, Calif., the steel, sheet-pile bulkhead failed in two places (see Fig. 1); that is, in 12 ft and 25 ft of water, more or less, respectively. As the City and the contractor disagreed on the cause of these failures, a suit was filed by the latter, which was tried in the Superior Court of Los Angeles County during the second half of 1932.

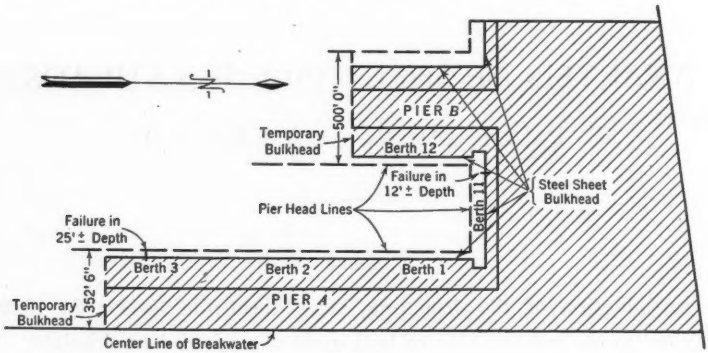


Fig. 1.

In substance the plaintiff claimed that the failure was due to faulty design and sea action on the toe slopes, while it was the defendant's contention that the failure was caused by faulty construction, particularly due to over-dredging the toe slopes and failure to place a preliminary fill behind the bulkhead.

At both places, the pushing out of the steel sheet-piling was directly attributable to the removal of material below the theoretical toe slope (Fig. 7) by whatever agency that caused it.

The toe slopes were undercut by a suction dredger as in ordinary side-slope work, which was a dangerous procedure in view of the importance of the toe material relative to the stability of the bulkheads.

On the other hand, a terrific storm occurred during the dredging operation, which demolished part of the break-water and caused considerable agitation in the harbor basin, and it was probably this evidence more than anything else that prompted a judgment substantially in favor of the contractor.

For the purpose of establishing facts in connection with this case the following tests were made: (1) Soil tests on samples from the fill area and from the channel; (2) tests to establish the depth of water standing in the fill area; (3) tests to establish the passive resistance of the fill; and (4) tests to establish the minimum interlock efficiency of steel sheet-piling.

Soil Tests

The soil tests were made in the City Laboratory in Long Beach, in accordance with the standards of the American Society for Testing Materials where such standards existed, and with methods adopted by the supervising

engineers, Charles T. Leeds, M. Am. Soc. C. E., Frederick J. Converse, Assoc. M. Am. Soc. C. E., and the writer. These tests were concerned particularly with the determination of the angle of repose "in the dry" and under water.

For the determination of the angle of repose in the dry, a standard inverted 12-in. slump cone was set on a smooth steel plate and filled with dry material. The cone was then lifted slowly, permitting the material to escape at the bottom (smaller) opening and to form a cone. Without exception these cones were very regular, and the angle of repose differed little at various locations.

Three methods were investigated to determine the angle of repose under water. In this connection, a water-tight container, about 2 by 2 by 2 ft, with a glass front, was used.

With the first method, a dry cone was produced in the empty container which was then gradually flooded until it was completely submerged. Complete saturation was indicated when air bubbles no longer escaped, when the water was siphoned off carefully and the angle of repose measured in at least three different places.

With the second method, saturated material was released through the large opening of a 6-in. slump cone into the filled container; the water was then siphoned off and the angle of repose measured as before.

Finally, with the third method, a 6-in. slump cone was set on the bottom of the empty container in an inverted position and was weighted down so as to make it water-tight at the bottom. The container was then filled to within ½ in. from the top of the cone and the latter was filled with dry material. The cone was then raised slowly until all the material had escaped at the bottom and had formed a cone. When the water was siphoned off, the angle of repose was measured as before.

As the last method consistently gave smaller angles of repose than the first and second methods, only the results due to Method 3 were recorded.

Standing Water in the Fill Area.—For the purpose of establishing the standing water level, two holes, roughly 30 ft in diameter at the surface and with the bottom at mean lower low water, were excavated and the water-surface elevation was recorded three times daily for a period of several weeks. This work was done in the fall of 1930. The observations showed a very small variation of the water-surface elevation from + 3.50 at a distance of 50 to 75 ft from the bulkhead.

Tests to Establish the Passive Resistance of the Fill

Test Equipment.—For the purpose of establishing the passive resistance of the fill, steel sheet-piling was driven in the fill area of Pier *B* (Fig. 1) so as to form a tank, as shown in Fig. 2.

The face of the tank, consisting of seven sheets, 28 ft long, and driven to a maximum of 12 ft into the ground, was free to deflect (fully 10 in.) within the yield of the copper seals. To each of three alternate sheets (see Fig. 2(*b*)), being convex to the outside, six ½-in. threaded rods were brazed, 3 ft on centers, so as to have the bottom rods coincident with the ground surface. The copper seals were carried to a depth of 7 ft, more or less (minimum, 5 ft), below the surface.

At a vertical distance of 15 ft above the ground surface, four of the seven sheets forming the test wall were anchored to the back-wall by means

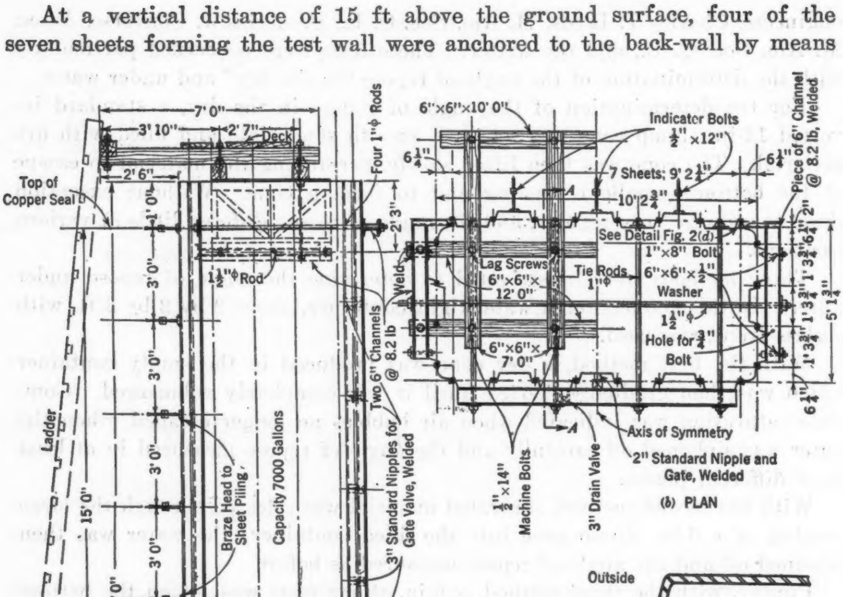


FIG. 2.

of 1-in. rods, but walers were omitted to prevent interference with slipping along the interlock. As sea water was the medium that produced pressure, the interlocks of the test wall were made partly water-tight through oakum and mastic, the first being loosely placed, not caulked, and serving to keep the mastic from squeezing through the joints.

Five 3-in. valves, 3 ft on centers, were welded to one of the sheets of the back-wall for the purpose of regulating the water surface during the test.

The deflections were measured by means of a transit set up 100 ft from the tank and sighting parallel to the test wall approximately flush with the free ends of the indicator rods. While the transit remained stationary, a nut was screwed on each indicator rod until its flat side was in line with the vertical hair of the transit. Each indicator nut was secured in its position by a lock nut. The distances on the indicator rods corresponding to the deflections were determined with a pair of dividers and were then sealed by means of a special rule reading to 0.01 in.

Prior to the test the fill in front and immediately adjacent to the tank was flooded with sea water and was kept wet during the tests. A test hole made with a 6-in. auger immediately before the final test showed the ground to be in a wet condition between the surface and the limit of capillary attraction which appeared to be about at Elevation + 4.50, or 1 ft above standing water.

Test Results.—The average sand in this location is believed to be closely represented by Sample *L* in Table 1; that is, the average specific weight was 2.70, and the percentage of voids, 50, which gives a weight per cubic foot of about 85 lb. The average moisture content in the sand was estimated at

TABLE 1.—TESTS ON SAMPLES OF FILL MATERIAL

Sample	Weight per cubic foot, dry and rodded, in pounds	Angle of Repose		Specific gravity	Percentage of voids
		Dry	Wet		
A.......	78.16	37°	30°, 29°, and 30°	2.70	50.72
B.......	101.56	35°	27°, 30° 30′, and 28°	2.70	39.75
C.......	102.42	35°	23°, 27°, and 26°	2.68	38.70
D.......	82.67	36°	30°, 30°, and 30°	2.72	51.25
E.......	78.05	37°	31°, 31°, and 31°	2.70	53.63
F.......	89.70	35°	27°, 30°, and 29°	2.70	46.71
G.......	76.52	39°	30°, 31°, and 30°	2.70	54.54
H.......	95.64	36°	30°, 28°, and 30°	2.70	43.20
I.......	89.85	36°	32°, 30°, and 32°	2.71	46.82
J.......	97.19	35°	31°, 34°, and 35°	2.60	40.05
K.......	87.65	36°	31°, 30°, and 31°	2.70	47.93
L.......	81.07	37°	31°, 30° 30′, and 31°	2.70	51.84
M.......	56.49	39°	35°, 36°, and 35°	2.62	65.41
N.......	75.79	37°	28°, 26°, and 27°	2.70	54.98

from 15 to 20% by weight, which means a 50% saturation, more or less. The effective weight of the material, offering passive resistance, was estimated at 100 lb per cu ft. The angle of internal friction (30°) was assumed to coincide with the angle of repose under water.

The active pressure below the ground surface is subject to estimation, since a theoretical analysis of such pressure is impossible, due particularly to the uncertainty of the hydrostatic pressure under the influence of percolation. The average fineness of the material at this location (Sample *L*, Fig. 3), permits a small percolation velocity, and the corresponding pressure on the sheet-piling was assumed to vary parabolically rather than linearly between the ground surface and the bottom of the sheet-piling, the latter coinciding approximately with standing water level. The parabolic distribution of pressure (in contradistinction to the usual assumpton of linear distribution with a zero

lateral pressure at the bottom of the sheet-piling) is believed more nearly to represent the true condition due to the relatively short duration of the test. The influence on the stability of the structure of this assumption, as compared to the linear distribution, is small.

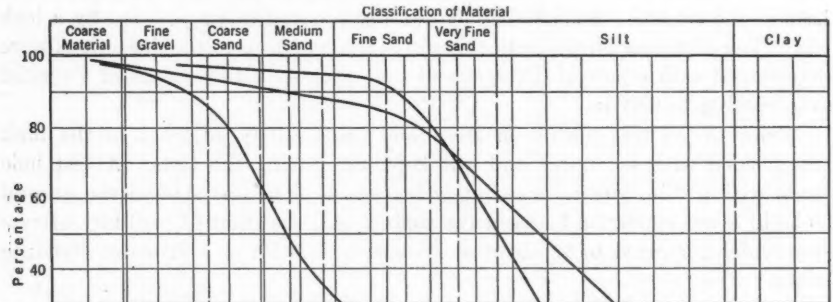


FIG. 3.

Under these assumptions the active forces are determined, the principal one being the water pressure between the upper support and the ground surface, which is known exactly. The active forces below the ground surface are not known exactly, but can be approximated with reasonable accuracy. The error that may be involved affects the ratio between the actual passive resistance and that based on Coulomb's formula, to a small degree.

Determination of Effectiveness Factor of Passive Resistance

The equilibrium conditions were analyzed for passive resistances of 100, 125, 150, and 175%, respectively, of the one due to Coulomb's formula for a frictionless wall. The Coulomb formula is:

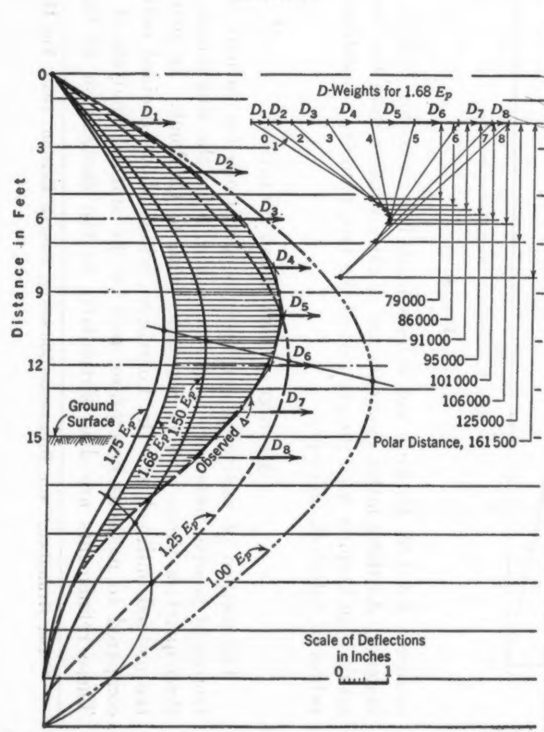
$$E_p = \frac{1}{2} \gamma t^2 \tan^2 \left(45 + \frac{\phi}{2}\right)$$

in which, E_p is the passive resistance of earth, in pounds; γ, the weight per cubic foot of sand; t, the penetration of sheet-piling, in feet; and ϕ, the angle of internal friction assumed to be equal to the angle of repose under water = 30 degrees.

The equilibrium conditions for each case were based on the interlocked section modulus, namely, $S = 9.30$ in.^{3} and a modulus of elasticity of $E = 29 \times 10^6$ lb per sq in. These elastic lines were finally plotted (see Table 2) to the same scale as the observed line, with the result that an effectiveness factor of 1.68 was found to represent the actual conditions most closely. The following values, per foot of wall, apply in the case of Table 2:

TABLE 2.—MOMENTS, DEFLECTIONS, AND EFFECTIVE SECTION MODULUS

Deflections



Moments, M, for 1.75 E_p, in foot-pounds	Unit Stress, $\frac{M}{S}$, in Pounds per Square Inch: Single sheet-pile	Unit Stress: Interlocked sheet-piling	Deflections, Δ, in Inches: Observed	Deflections: Computed	Effective $E\,I$ (percentage)	Effective I (percentage)	Effective E (Percentage)	Effective E In pounds per square inch	Effective S, in inches³	Fiber stress f, in pounds per square inch
3 200	9 200	4 130	0.68	0.33	39.50	40.00	99.00	28.75×10^6	3.72	10 350
9 425	27 200	12 150	1.91	0.93	43.00	44.50	96.50	28.00×10^6	4.14	27 300
15 000	43 250	19 350	3.00	1.44	45.50	50.00	91.25	26.50×10^6	4.65	38 700
19 200	55 400	24 750	3.80	1.82	47.50	56.25	84.50	24.50×10^6	5.23	44 000
21 600	62 350	27 900	4.16	2.03	50.50	62.25	81.00	23.50×10^6	5.80	44 700
21 900	63 200	28 300	4.15	2.07	53.00	64.00	82.75	24.00×10^6	5.95	44 200
19 200	55 400	24 750	3.74	1.93	62.50	67.25	93.00	27.00×10^6	6.28	36 700
12 900	37 200	16 630	3.00	1.59	80.75	82.25	98.00	28.50×10^6	7.65	20 300
3 300	9 525	4 260	....	1.16	(99)	(100)	(99)	28.75×10^6	(9.30)	(4 270)
−8 220	23 700	10 600	....	0.70	....	....	....	....	....	....
−16 200	46 750	20 900	....	0.33	....	....	....	....	....	....
−17 700	51 100	22 800	....	0.09	....	....	....	....	....	....
−8 220	23 700	10 600	....	−0.05	....	....	....	....	....	....
±0	±0	±0	....	−0.09	....	....	....	....	....	....

For a single sheet-pile,

$$S_{x\text{-}x} = 4.17 \text{ in.}^3$$
$$I_{x\text{-}x} = 2.94 \times 4.17 = 12.25 \text{ in.}^4$$
$$EI_{x\text{-}x} = 12.25 \times 29 \times 10^6 = 355 \times 10^6 \text{ in}^2\text{-lb.}$$

For interlocked sheet-piling,

$$S_{n\text{-}n} = 9.30 \text{ in.}^3$$
$$I_{n\text{-}n} = 2.94 \times 9.30 = 27.35 \text{ in.}^4$$
$$EI_{n\text{-}n} = 27.35 \times 29 \times 10^6 = 793 \times 10^6 \text{ in}^2\text{-lb.}$$

The interlock efficiency—that is, the effective moment of inertia of the wall about the axis of symmetry—was found by interpolating the *D*-weights for 1.68 E_p and by determining the polar distances of the string polygon (Table 2), which coincides with the observed deflection curve. The effective index of rigidity, $E\,I$, in percentage of its maximum value for $E = 29 \times 10^6$ lb per sq in. and $I = 27.35$ in.^{4}, is found to be:

$$E\,I = \frac{\text{Polar distance} \times 100}{H_{\text{max.}}} \quad \text{(1)}$$

and the effective section modulus was found by trial and error; that is, by determining corresponding *E*-values which, in turn, will check the stress-strain diagram of the average of seven samples as shown in Fig. 4. The

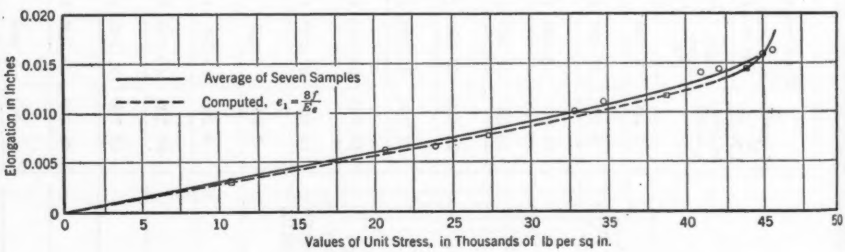


FIG. 4.

moments for 1.68 E_p were not interpolated, but those for 1.75 E_p were used instead. As these latter moments are slightly smaller than the first ones, the result is in keeping with the purpose of the test, namely, to establish minimum values of interlock efficiency.

METHOD OF GRAPHICAL ANALYSIS

The method of analyzing equilibrium conditions for various effectiveness factors of passive resistance is shown in Fig. 5 for the single case of one and three-quarters times passive resistance due to Coulomb. In principle it is the method outlined by Dr. Lohmeyer[3], which shows decided refinements as compared to methods heretofore used in the statical analysis of bulkheads. These refinements are due particularly to the recognition of the effect of

[3] "Der Grundbau," von Brennecke-Lohmeyer, Fourth Edition, 1930, Vol. II. p. 77 *et seq.*

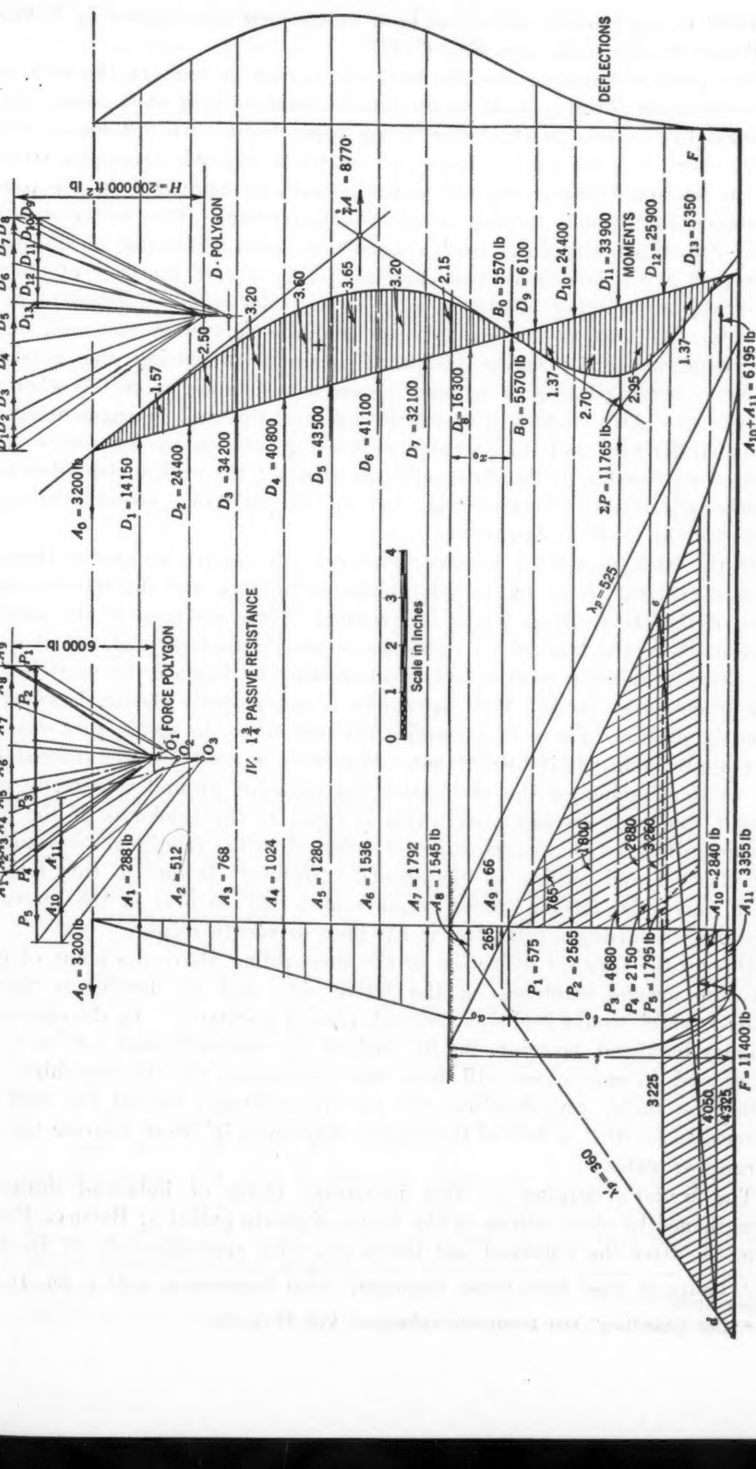

FORCE POLYGON
D - POLYGON
DEFLECTIONS
MOMENTS
IV. 1 3/4 PASSIVE RESISTANCE
Scale in Inches
H = 200000 ft² lb
6000 lb
A_0 = 3200 lb
B_0 = 5570 lb
ΣP = 11765 lb
F = 11400 lb
$A_{10}+A_{11}$ = 6195 lb
λ_p = 525
λ_a = 360

FIG. 5.

restraint in the ground, which has been comprehensively treated by Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E.[3]

The passive resistance on the active side, that is, back of the wall, is of the same order of magnitude as the one in front and, in most cases, may be assumed to be equal to the latter as an upper limit. An opinion is extant[4] to the effect that the passive resistance on the active side is greatly reduced, due to friction between the soil and the wall, as compared to the passive resistance that would develop without wall friction. The writer does not agree with this viewpoint. Such a condition could exist only as long as no movement of the bottom part of the sheet-piling toward the back of the wall occurred; or, in other words, as long as the piles were subjected to active pressure. As soon as deflection begins and the bottom of the wall, or the part adjacent to it, moves backward (thereby displacing the particles) the forces getting into play are due to passive resistance, as on the other side of the wall. Any friction between the soil and the wall is augmenting this passive resistance, and the principal difference between the two cases is the angle of inclination of the sheet-piling in front of the wall, which (due to its "overhang") tends to increase the slope of the resistance vector, while back of the wall it tends to decrease it.

If the back requires a smaller allowance for passive resistance than the front of the wall, it is due to this difference in slope and the relative movement of the sheet-piling, which is governed by the distance of the point of rotation above the bottom. To produce a certain passive resistance at a certain depth requires a certain movement tending to displace the particles and thereby compress them. This movement is not entirely elastic, however; it is partly plastic. There is a certain lost motion, so to speak, that does not meet appreciable passive resistance. However, it must be remembered that due to the driving of the sheet-piles the adjacent ground is already compressed, due to a displacement which is equal to the thickness of the piles. The magnitude of this compression is reflected in the frictional resistance to both driving and pulling of the sheets. "Refusal" is due to this friction, which can only develop under compression; and so also is the enormous resistance to raising a pile after it has been driven to refusal.

Due to the forward deflection of the sheet-piling above the point of rotation, the ground stretches on the active side, and its density is thereby reduced, which would lead to a reduced, passive resistance. In the case of an ordinary bulkhead, however, the fill back of the sheet-pile wall acts as a surcharge and, in most cases, will more than compensate for the stretching. As to the test under consideration, the passive resistance behind the wall was assumed to be 70% or less of the passive resistance in front, because the surcharge was water.

The writer's opinion on this important phase of bulkhead design is strengthened by observations of the shape of sheets pulled at Berth 3, Pier *A* (Fig. 1), after the bulkhead had blown out with approximately 27 ft of fill

[3] "Design of Steel Sheet-Piling Bulkheads," *Civil Engineering*, Vol. 3, No. 11, November, 1933.

[4] "Der Grundbau" von Brennecke-Lohmeyer, Vol. II, p. 83.

behind it; and of the shape of sheets used in the test (Fig. 6). As to piles that were pulled after the failure at Berth 3 (Fig. 1), a statical analysis, for the original conditions, shows no restraint (Fig. 7), while the sheets, after pulling, showed decided counterflexure, which would have been impossible under the contention[4] that passive resistance on the active side is only 18% of that in front of the wall.

Returning to the statical analysis, the next step will be to ascertain whether or not there is a restraint. For this purpose subtract the active from the passive pressure area and extend the resulting passive pressure area; that

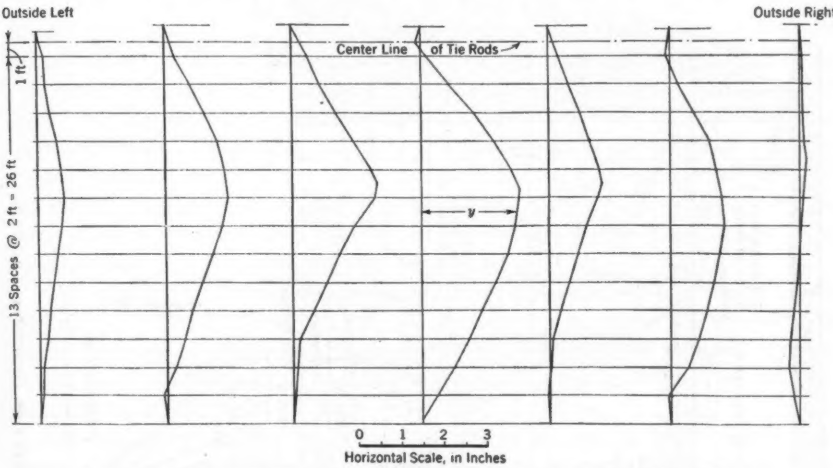


FIG. 6.—PERMANENT DEFORMATION OF TEST PILES FACING TEST WALL.

is, the λ_p–λ_a line indefinitely downward (Fig. 5(*a*)), divides the pressure area conveniently, and combines the corresponding active and passive forces in a force polygon. Then draw the moment diagram (Fig. 5(*b*)), which will be S-shaped, and extend it far enough downward to cover the entire known or probable penetration.

The minimum, theoretical penetration is then immediately found by means of a line through the upper support point and tangent to the moment line. The depth of the point of contact below the natural ground surface is then equal to the minimum theoretical penetration.

The next step will be to determine the practical minimum penetration, in which case the passive resistance at the bottom of the sheet-piling must be zero. At this penetration there is no restraint, but any additional penetration will cause part, and finally full, restraint.

In graphic statics, the three equilibrium conditions — $\Sigma H = 0$; $\Sigma V = 0$; and, $\Sigma M = 0$ — are satisfied if both the polygons close, the force polygon satisfying $\Sigma H = 0$ and $\Sigma V = 0$, and the string polygon being such that the sum of all the moments about an arbitrary point is zero ($\Sigma M = 0$).

If the penetration is given, as in this case, the problem is to find the most unfavorable load distribution for which equilibrium exists. If the penetra-

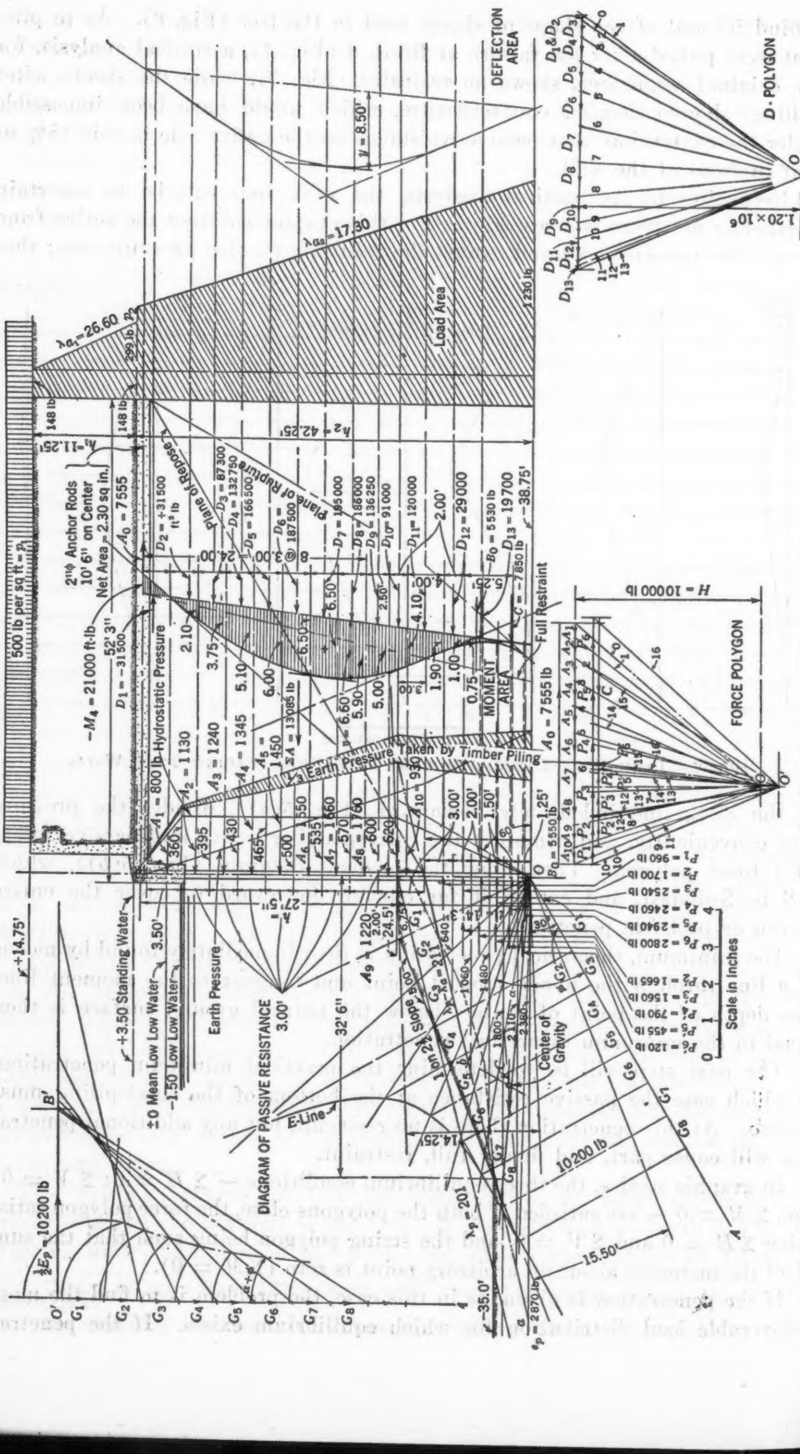


FIG. 7.—EXAMPLE OF BULKHEAD ANALYSIS.

tion is such as to permit restraint to develop, the problem is again facilitated in using a single force, which, in this case, must go through the point of rotation of the sheet-piling; that is, the point some distance above the bottom which, during deflection, remains stationary. Force F (Fig. 5) again must be found by trial; it represents the resultant resistance of the cross-hatched area behind the sheet-piling. The final load distribution is found by deducting the cross-hatched resistance area on the active side from that on the passive side, and by drawing Line d'-e' so as to balance the differential areas above and below this line.

In Fig. 7 a section through the bulkhead at Berth 3 (Fig. 1) at the place of failure is shown. A 1-ft width of wall is considered. The method of analysis as previously outlined is used to ascertain the safety of the structure as originally designed. The analysis is based on the following properties, established by the foregoing tests and by supplementary tests on soil from the channel bottom; Angle of repose, dry, = 35° and, under water, = 30°; weight of dry material = 90 lb per cu ft; weight of material under water = 52 lb per cu ft; average voids = 40%; weight of sea water, W, = 64 lb per cu ft; level of water, standing in fill = Elevation + 3.50; effective lowest low water = Elevation − 1.50; and effective hydrostatic head = 5 ft.

The analysis is further based on the following assumptions: Surcharge on surface, p_1 = 500 lb per sq ft; the effectiveness factor of passive resistance of natural ground and a rough wall = 2; and the angle of internal friction of the sand is equal to the angle of repose and is constant, that is, independent of the depth.

The timber piles (not shown in Fig. 7), at least 16 in. in diameter, that support the relieving platform, take one-third the active earth pressure. The spacing of these piles in the first row, which is 2 ft 6 in. back of the steel sheet-piling, is 2 ft 11 in. (average) from center to center. This assumption is believed to be conservative, as nearly one-third the volume of the active prism is retained by the timber piling without the aid of arching.

The influence of the reinforced concrete encasement beam at the top of the sheet-piling was neglected, but the restraint of the sheet-piling due to the part that was encased in the cantilever end of the reinforced concrete relieving platform, was considered. The critical section of the platform has an over-all depth of 22 in. and the center line of the reinforcing steel, consisting of $\frac{3}{4}$-in. square bars, 8 in. on centers, is 19 in. from the top.

The load area due to earth pressure was determined by means of the relations, $\lambda_a = \gamma \tan^2 \left(45 - \dfrac{\phi}{2}\right)$ and $e_a = \dfrac{\lambda_a\, p}{\gamma}$. The influence on the bulkhead of earth and surcharge above the relieving platform was included.

The minimum passive resistance, E_p, of the material below the theoretical toe slope was determined graphically by means of Culmann's E-line method at 20 400, or 64% of the minimum passive resistance for a level ground surface at Elevation — 24.50.

The analysis shows that the lower end of the bulkhead is not restrained and that it is a case of practical, minimum penetration. The findings may be summarized as follows:

Net passive resistance, $E_p = 13\,400$ lb
Lower reaction, $B_0 = 5\,530$ lb
Safety against push-out,

$$S_p = \frac{13\,400}{5\,530} = 2.42$$

Average stress in anchor rod and bars,

$$f_a = \frac{10.50 \times 7\,555}{2.30 \times 20 \times 0.39} = 7\,850 \text{ lb per sq in.}$$

Maximum fiber stress in steel sheet-piling used,

$$f_s = \frac{6.60 \times 10\,000 \times 12}{25.86} = \pm\ 30\,600 \text{ lb per sq in.}$$

Stresses in critical concrete section:

Maximum compression in concrete = 460 lb per sq in.
Maximum tension in reinforcing steel = 17 400 lb per sq in.

Maximum deflection of steel sheet-piling,

$$\Delta\ (\text{max.}) = \frac{1.20 \times 10^6 \times 8.50 \times 12^3}{29 \times 10^6 \times 25.86 \times 5.687} = 4.13 \text{ in.}$$

The average of twenty-eight tests of specimens cut at random from the steel sheet-piling showed a yield point of 45 782 lb per sq in. and a breaking strength of 75 510 lb per sq in.

If the distance from the original ground surface at the center line of the sheet-piling to the zero point of the load area is termed a_0, and the distance from there to the bottom is t_0, then it may be shown[5] that the total penetration is,

$$t = a + 1.20\ t_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The distance, X_0 (Fig. 7), which is the point of zero moment below the original ground surface, reaches a minimum value for full restraint, and the same is given at approximately 0.10 h_0, provided the angle of repose of the ground under water is at least 30 degrees. The height of the top of the fill above the natural ground at the center line of the sheet-piling is denoted by h_0.

From the equation for the restraining or negative moment and from Equation (2), a formula for the penetration, t_f, for full restraint is obtained, which when simplified reads,

$$t_f = 1.60\ a_0 - 0.06\ h_0 + 1.20 \sqrt{\frac{6\ B_0}{2\ \lambda_p - \lambda_a}} \ldots\ldots\ldots(3)$$

[5] "Der Grundbau," von Brennecke-Lohmeyer, Vol. II, p. 85.

and, in which, in order to get t, in feet, a_0 and h_0 are introduced, in feet, B_0, in pounds, and,

$$\lambda_p = \gamma \tan^2 \left(45 + \frac{\phi}{2}\right) \qquad (4)$$

and,

$$\lambda_a = \gamma \tan^2 \left(45 - \frac{\phi}{2}\right) \qquad (5)$$

Equation (3) represents a valuable contribution to the art of bulkhead design, as it eliminates all the trial work previously described as far as the condition of full restraint is concerned. The deflections were determined graphically by Mohr's method; that is, by using the moment area as load area for a constant index of rigidity, EI.

From a standpoint of safety only, full restraint is always desirable, but the corresponding penetration is not necessarily the most economical one. The section modulus, S, of steel sheet-pile sections varies approximately with the square of the weight, G, per unit length. Thus,

$$\frac{S_1}{S_2} = \frac{G^2_1}{G^2_2}, \text{ or } \frac{G_1}{G_2} = \sqrt{\frac{S_1}{S_2}} \qquad (6)$$

On the other hand, the required section modulus, based on a limiting fiber stress, is directly proportional to the maximum moment. Therefore,

$$\frac{G_1}{G_2} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{\zeta_1}{\zeta_2}} \qquad (7)$$

if $M_1 = H_0\,\zeta_1$ and $M_2 = H_0\,\zeta_2$.

If the length of sheets in a wall with part restraint is l_1, and that for full restraint is l_2, the required theoretical steel weight per unit length of l_2 in percentage of l_1 is,

$$G = 100\,\frac{l_2}{l_1}\sqrt{\frac{\zeta_2}{\zeta_1}} \qquad (8)$$

which is not necessarily less than 100. The practical economy factor may differ from the theoretical because it is governed by the available sections. This is illustrated in the example shown in Fig. 7. The section used was such as to resist a maximum moment of $6.60 \times 10\,000 \times 12 = 792\,000$ in-lb, and the total length of sheet-piling was 42 ft, of which, 27.75 ft is above the ground and 14.25 ft is penetration. To produce full restraint 19.40 ft of penetration would be required, which would make a total length of sheets of $27.75 + 19.40 = 47.15$ ft, and which would reduce the maximum moment to $\frac{4.60}{6.60} \times 792\,000 = 552\,000$ in-lb.

To keep within the same stress, namely, 30 600 lb per sq in., the section modulus of the next lighter section would be insufficient making the use of

the original section for both cases necessary. Instead of a saving, there would actually be a greater cost for total restraint, due to additional length.

It must be understood that the load distribution below the original ground surface, shown in Fig. 7, is merely an approximation of the actual distribution. The latter could easily be arrived at by mere examination and trial, but would be of no advantage from the standpoint of safety, because the triangular distribution as used in the analysis gives greater maximum moments and calls for greater penetration.

The bulkhead, as shown in Fig. 7, differs from the test wall (Fig. 2) in that the sheets are partly restrained at the top due to a reinforced concrete encasement and in that the ground in front of the bulkhead is dredged to a 1 on $2\frac{1}{2}$ slope, leaving a berm of about 6.75 ft.

In view of the fact that an effectiveness factor of 1.68 was established by the test in the artificial fill, the assumption of a factor of 2.00 for natural ground is believed to be safe even for sloping ground.

Analytical Determination of Interlock Efficiency

The observed, elastic line is closely approximated by two hyperbolic spirals with a common point and tangent at the point of minimum radius of curvature; that is, maximum moment. This curve becomes faulty near the top support where it is of small importance, as the radius of curvature remains finite. However, it closely checks the elastic line in the vicinity of minimum radius of curvature.

The equation of the upper spiral going through the upper point of support is,

$$\sqrt{(u_1 - z)^2 + (v_1 + y)^2}\left\{\text{arc}\ \tan\frac{v_1 + y}{u_1 - z} + \alpha_1\right\} = a \ldots\ldots\ldots (9)$$

and the equation of the lower spiral, going through the observed point at the ground surface, is:

$$\sqrt{(u_2 + z)^2 + (v_2 + y)^2}\left\{\text{arc}\ \tan\frac{v_2 + y}{u_2 + z} + \alpha_2\right\} = a \ldots\ldots\ldots (10)$$

The constants, u_1, u_2, v_1, v_2, α_1, α_2, and a, are determined from the observed elastic line, and the tangent in any point may then be determined for the upper curve through,

$$\frac{dy}{dz} = \frac{a\,(u_1 - z) - (v_1 + y)\sqrt{(u_1 - z)^2 + (v_1 + y)^2}}{a\,(v_1 + y) + (u_1 + z)\sqrt{(u_1 - z)^2 + (v_1 + y)^2}} \ldots\ldots (11)$$

and through,

$$\frac{dy}{dz} = \frac{-\,a(u_2 + z) + (v_2 + y)\sqrt{(u_2 + z)^2 + (v_2 + y)^2}}{a\,(v_2 + y) + (u_2 + z)\sqrt{(u_2 + z)^2 + (v_2 + y)^2}} \ldots\ldots (12)$$

for the lower curve.

Further, the radius of curvature for the upper curve is expressed as:

$$\rho_1 = \frac{\sqrt{(u_1 - z)^2 + (v_1 + y)^2}}{\sin^3\left\{\text{arc } \tan \dfrac{a}{\sqrt{(u_1 - z)^2 + (v_1 + y)^2}}\right\}} \quad \text{..........(13)}$$

and for the lower curve,

$$\rho_2 = \frac{\sqrt{(u_2 + z)^2 + (v_2 + y)^2}}{\sin^3\left\{\text{arc } \tan \dfrac{a}{\sqrt{(u_2 + z)^2 + (v_2 + y)^2}}\right\}} \quad \text{..........(14)}$$

The common point follows from $\rho_1 = \rho_2$, which is the case for $z = 10.17$ ft. As the moment, $M = A_o z - \dfrac{64\ z^3}{6}$, must be a maximum at this point: $\dfrac{dM}{dz} = 0 = A_o - 32\ z^2$; and $A_o = 3\,310$ lb. This reaction checks very closely the one determined graphically for 1.68 E_p by interpolation.

Fig. 8 shows ρ as a function of z. The dotted line at the lower end indicates a correction, based on the boundary condition that ρ must become

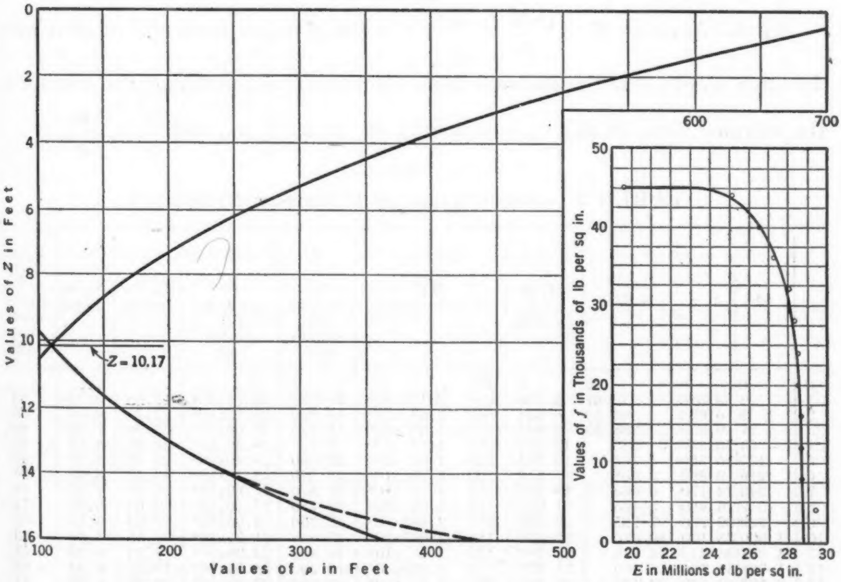


FIG. 8.—RADIUS OF CURVATURE.

FIG. 9.—RELATION BETWEEN f AND E.

infinite for $z = 18 \pm$. A horizontal line through $z = 18$, therefore, should be asymptotic to the ρ-curve. It is believed that these corrected ρ-values are a better approximation of the actual condition, and it is for this reason that they were used in the computation of the interlock efficiency. The lower

spiral is subject to greater error than the upper one, because it does not go through the origin of co-ordinates and is governed by fewer check points.

In Fig. 9, the fiber stress, f, is shown as a function of the modulus of elasticity, E, based on the average of seven tests. Knowing ρ and M as functions of z, the next step is to find a relation between ρ and E, which follows from the simplified equation of the elastic line,

$$\frac{d^2y}{dx^2} = \frac{1}{\rho} = \frac{M}{EI} \qquad (15)$$

Notwithstanding the fact that the test involves material deflections which are not negligible as compared to the over-all dimensions of the wall, the error in using the approximate term, $\frac{d^2y}{dz^2}$, for $\frac{1}{\rho}$, instead of $\frac{\frac{d^2y}{dz^2}}{\left[1+\left(\frac{dy}{dz}\right)^2\right]^{\frac{3}{2}}}$, is small since $\frac{dy}{dz}$ varies within a range of 0.07 which means that the maximum value of $\left(\frac{dy}{dz}\right)^2$ is about 0.005.

In the formula, $E = \frac{\rho M}{I} = \frac{\rho f}{e}$, e is the distance from the neutral axis. In other words, e is the distance from the axis of symmetry of the section to the extreme fiber, in this case, $e = 2\frac{15}{16}$ in. $= 2.937$ in., and $\frac{f}{E} = \frac{e}{\rho}$.

TABLE 3.—Computation of Interlock Efficiency

z	z^3	$A_o\,z$	$10.67z^3$	M, in foot-pounds	EI, in foot²-pounds	f	E	I_e, in inches	S_e, in inches	Interlock efficiency (percentage)
						In pounds per square inch	In pounds per square inch			
0	0	0	0	0	751	......	....	......	....	.....
1	1	3 310	10.67	3 299	639 2.172×10^6	10 680	28.74×10^6	10.88	3.72	40
2	8	6 620	85.50	6 534	540 3.53×10^6	13 020	28.72×10^6	17.70	6.05	65
3	27	9 930	288.50	9 641	453 4.37×10^6	15 475	28.66×10^6	21.96	7.43	80
4	64	13 240	683	12 557	380 4.77×10^6	18 425	28.59×10^6	24.02	8.18	88
5	125	16 550	1 335	15 215	316 4.81×10^6	22 050	28.49×10^6	24.32	8.27	89
6	216	19 860	2 308	17 552	262 4.60×10^6	26 420	28.29×10^6	23.42	8.00	86
7	343	23 170	3 663	19 507	214 4.19×10^6	31 870	27.89×10^6	21.63	7.35	79
8	512	26 480	5 465	21 015	175 3.68×10^6	37 780	27.10×10^6	19.60	6.70	72
9	729	29 790	7 785	22 005	141 3.105×10^6	43 250	24.95×10^6	17.95	6.14	66
10	1 000	33 100	10 670	22 430	114 2.557×10^6	45 600	21.25×10^6	17.35	5.86	63
11	1 331	36 410	14 220	22 190	132 2.93×10^6	44 450	23.98×10^6	17.60	5.95	64
12	1 728	39 720	18 450	21 270	162 3.446×10^6	40 200	26.55×10^6	18.68	6.32	68
13	2 197	43 030	23 470	19 560	200 3.91×10^6	33 880	27.68×10^6	20.35	6.88	74
14	2 746	46 340	29 300	17 040	248 4.225×10^6	27 830	28.20×10^6	21.60	7.35	79
15	3 375	49 650	36 040	13 610	320 4.355×10^6	21 760	28.48×10^6	22.05	7.53	81

In Table 3 the interlock efficiency and the effective section modulus are computed and the resulting fiber stresses are shown in Fig. 10. It is interesting to note that near the upper support the effective section modulus is

smaller than the one for the single sheet which can only be interpreted as the effect of flattening out, due to the anchor pull and the lack of a waling and of lateral support.

The same effect probably accounts for the rapid drop of the interlock efficiency toward the point of minimum radius of curvature since the sheets did not have any lateral support, because the wall is short; they would tend

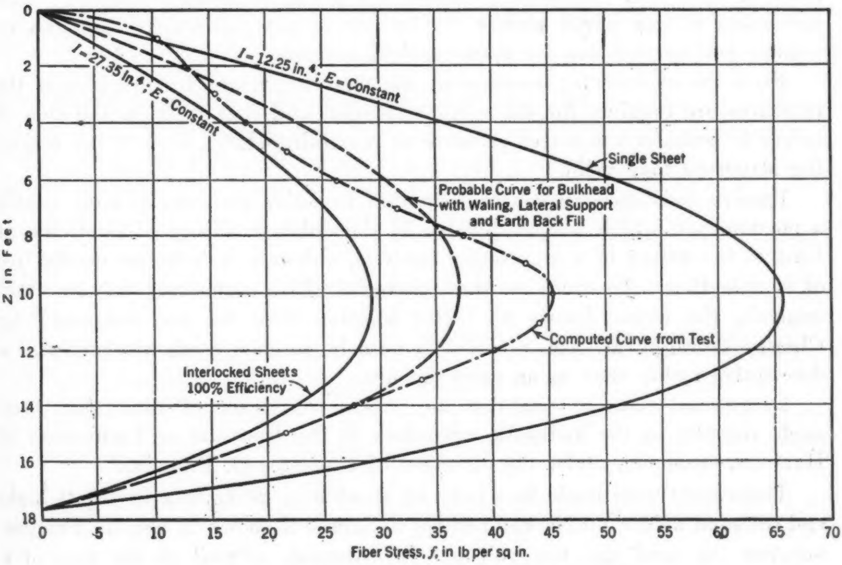


FIG. 10.—FIBER STRESSES COMPUTED FROM TEST.

to flatten due to lateral thrust. As the ground is approached and penetrated, the lateral freedom disappears rapidly and the interlock efficiency only shows the effect of longitudinal slippage. In fairness to the manufacturers of this steel sheet-piling, the writer has indicated a dotted line in Fig. 10, which he believes would be a close approximation of the actual interlock efficiency of a bulkhead of equal height as the one used in the test, but with waling at the tie-backs, lateral support, and, last but not least, back-filled with saturated earth instead of water.

In the case of a sheet-pile wall driven in natural ground and held at the top by a concrete beam (Fig. 7), or otherwise, but so as positively to prevent slippage, the section modulus for the interlocked sheets as given by the manufacturer could be used without hesitancy.

PASSIVE RESISTANCE AS A FUNCTION OF BOTH z AND y

In the foregoing, both active pressure and passive resistance of earth were treated as a statically determinate problem in accordance with the present theory. In other words, earth was treated like a fluid of smaller or greater unit weight than earth for active pressure and passive resistance, respectively.

Evidently, this is not the true condition, but is an approximation that may lead to satisfactory results for practical purposes if properly applied. In the case of active pressure on an unyielding wall, it is an exact interpretation, because then the problem is statically determinate.

The pressures for this condition are maximum and decrease as the wall begins to yield, due to a stretch in the grain structure; and, finally, they reach a minimum immediately before the grain structure collapses. This movement is only partly elastic (in its first stage) and quickly becomes irregular and too complex for mathematical analysis.

From an engineering standpoint, the most important feature is that the pressures are maxima for the static condition and that it is on the side of safety to consider the active pressure as remaining static, even if the retaining structure may yield.

Passive resistance, in contradistinction to active pressure, is more nearly a phenomenon which is approachable by the ordinary theory of elasticity, at least to the extent of a qualitative analysis, although it is by no means free of complexities. To overcome these obstacles which stubbornly defy an exact analysis, the writer knows no better solution than the one suggested[6] by Charles Terzaghi, M. Am. Soc. C. E., namely, to class earth mechanics as a descriptive rather than as an exact science.

Large-sized tests to establish the passive resistance of sand have been made recently in the hydraulic laboratory of the Institute of Technology of Hanover, Germany, under the direction of Professor O. Franzius.[7]

These tests were made in a box, 6.6 ft wide by 23 ft long by 6.6 ft high, and the sand used would be comparable to Sample J shown in Fig. 3. Friction between the sand and the wall was investigated, as well as the case of a frictionless wall, the latter being produced by letting the wall rise (exactly counterbalanced) under the influence of friction. Values of E_p for various heights of fill are shown as functions of y, which is the horizontal movement of the wall; and likewise for the coefficient of friction, tan δ, between the sand and the wall; the latter, however, is in too small a scale to be used quantitatively. In all cases the sand was filled in in thin layers and tamped. The outstanding result of all the tests is the fact that with or without wall friction, y is a steady function of E_p and that the coefficient of friction between sand and wall decreases with the depth.

The relation between the movement, y, of the wall and E_p is satisfactorily expressed by,

$$y = f_1 E_p + f_2 E^{(n_o)}{}_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which, f_1 and f_2 are constants for one and the same height of fill $(z - h)$, and n_o is an exponent which, according to Professor Terzaghi[8] is approximately 2, which checks the value found by Franzius for wet sand.

[6] "Erdbaumechanik auf bodenphysikalischer Grundlage," von Charles Terzaghi, Franz Deutike, 1925.

[7] *Der Bauingenieur*, 9 Jahrgang, 1928, Hefte 43 und 44.

[8] "Erdbaumechanik," by Charles Terzaghi, p. 332.

For $y = 0$ there is active (static) pressure only; that is, $E_p = E_0$. Therefore, $0 = f_1 + f_2 E_0$, and,

$$E_0 = - \frac{f_1}{f_2} = \frac{\gamma \, \xi_0 \, (z - h)^2}{2} \text{ .... (17)}$$

in which, ξ_0, according to Professor Terzaghi,[9] is of the order of magnitude of 0.42.

Introducing Equation (17) into Equation (16),

$$y = f_2 \, (- E_0 \, E_p + E_p^2) \text{ .... (18)}$$

from which, f_2 may be determined for values of y and E_p obtained by test. The value, f_1, then follows from Equation (17) and, obviously, is negative.

Factors f_1 and f_2 can be expressed as steady functions of $(z - h)$, or, in general terms,

$$f_1 = - \frac{a}{(z - h)^m} \text{ .... (19)}$$

and,

$$f_2 = \frac{b}{(z - h)^n} \text{ .... (20)}$$

Solving Equation (16) for E_p and making $n_0 = 2$:

$$E_p = \frac{- f_1 + \sqrt{f_1^2 + 4 f_2 y}}{2 f_2} \text{ .... (21)}$$

and, because $dE_p = e_p \, dz$, or $\frac{d E_p}{dz} = e_p$:

$$e_p = \left[f_2 \left(- \frac{d f_1}{d z} + \frac{f_1 \frac{d f_1}{dz} + 2 y \frac{d f_2}{dz}}{\sqrt{f^2 + 4 f_2 y}} \right) - \left(f_1 + \sqrt{f_1^2 + 4 f_2 y} \right) \frac{d f_2}{dz} \right] \frac{1}{2 f_2^2} \text{ .... (22)}$$

Introducing Equations (19) and (20), and differentiating:

$$e_p = \left\{ \frac{b}{(z - h)^n} \left[- \frac{a m}{(z - h)^{m+1}} + \frac{- \frac{a^2 m}{(z - h)^{2m+1}} - 2 y \frac{b_n}{(z - h)^{n+1}}}{\sqrt{f_1^2 + 4 f_2 y}} \right] + \left[\frac{a}{(z - h)^m} + \sqrt{f_1^2 + 4 f_2 y} \right] \frac{b n}{(z - h)^{n+1}} \right\} \frac{(z - h)^{2n}}{2 b^2} \text{ ...... (23)}$$

or,

$$e_p = \frac{a \, (n - m) \, (z - h)^{(n-m)-1}}{2 b} \left\{ 1 + \frac{a}{R \, (z - h)^m} \right\} + \frac{n y}{(z - h) R} \text{ ... (24)}$$

in which, $R = \sqrt{\frac{a^2}{(z - h)^{2m}} + \frac{4 b y}{(z - h)^n}}$.

[9] "Erdbaumechanik," by Charles Terzaghi, pp. 187 and 195.

From the aforementioned tests by Professor Franzius and, particularly, for $y = 100$ mm $= 0.328$ ft; for wet sand and a rough wall, the following numerical values were found, based on the assumption that the passive resistance, E_p, as determined by test, is due to a width of 6.6 ft (2 m): $a = 10.00 \times 10^{-6}$; $b = 0.476 \times 10^{-6}$; $m = 0.535$; and, $n = 2.535$.

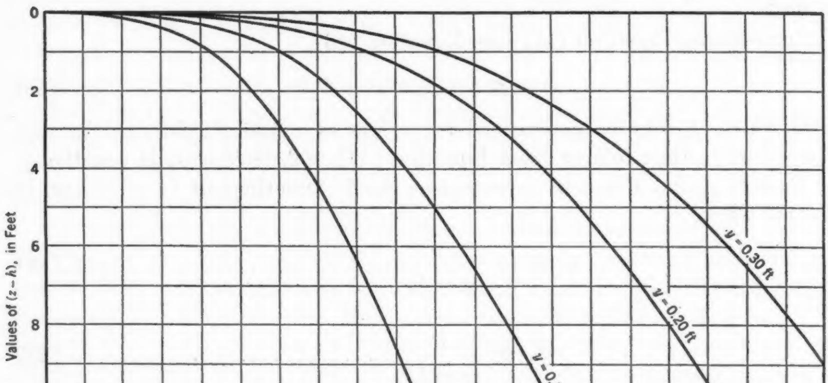


FIG. 11.—DISTRIBUTION OF SPECIFIC PASSIVE RESISTANCE, e_p.

The evaluation of Equation (24) is shown in Fig. 11. The first term in Equation (24) represents principally the influence of active pressure, and a variation of y only affects it through the root in the denominator. For $y = 0$, the term in the bracket becomes 2; then, $e_p = e_a$, and is due to active pressure only.

With sufficient accuracy, Equation (24) may be written in a simpler form, as follows:

$$e_p = \left(\gamma\, \xi_0 - 23 \sqrt[10]{y}\right)(z - h) + \frac{n y}{(z - h) R} \quad \ldots\ldots\ldots\ldots (25)$$

Remembering that $n - m = 2$, or $n = m + 2$, the last term may be written as follows:

$$\frac{n \sqrt{y}}{2 \sqrt{b}} \left\{ (z - h)^{0.5 m} \left[1 + \frac{a^2}{4 b y} (z - h)^{2-m} \right]^{-\frac{1}{2}} \right\}$$

It now remains to determine whether the term, $\left[1 + \dfrac{a^2}{4 b y} (z - h)^{2-m} \right]^{-\frac{1}{2}}$,

is convergent for such values of y as are of practical importance, thus:

$$\left[1 + \frac{a^2}{4\,by}(z-h)^{2-m}\right]^{-\frac{1}{2}} = 1 - \frac{a^2}{2 \times 4\,by}(z-h)^{2-m} + \frac{3}{8}\left(\frac{a^2}{4\,by}\right)^2 (z-h)^{4-2m}$$

$$- \frac{15}{48}\left(\frac{a^2}{4\,by}\right)^3 (z-h)^{6-3M} + \ldots$$

Introducing values for a and b, $\frac{a^2}{4\,b} = 0.0000525$, and:

$$\left[1 + \frac{a^2}{4\,by}(z-h)^{2-m}\right]^{-\frac{1}{2}} = 1 - \frac{0.0000525}{2\,y}(z-h)^{2-m}$$

$$+ \frac{3}{8}\left(\frac{0.0000525}{y}\right)^2 (z-h)^{4-2m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Movements of y much less than 0.01 ft are not believed to be of practical interest as they are likely to be partly plastic; their effect is small and difficult to observe. For $y = 0.01$, the right-hand side of Equation (26) becomes:

$$1 - 0.002625\,(z-h)^{2-m} + 0.000002582\,(z-h)^{4-2m} - \ldots$$

Multiplying by $(z-h)^{0.5m}$:

$$(z-h)^{0.5m} - 0.002625\,(z-h)^{v} + 0.000002582\,(z-h)^{2v} \ldots$$

in which $v = m - 0.5\,m^2$. For $(z-h) = 1$, the decrement due to the second and third terms is roughly $\frac{1}{4}\%$ of the first and for $(z-h) = 20$, it is about $\frac{3}{8}\%$ of the first; which means that, with close approximation,

$$e_p = (\gamma\,\xi_0 - 23\,\sqrt[10]{y})\,(z-h) + \frac{n\,\sqrt{y}\,(z-h)^{0.5m}}{2\,\sqrt{b}} \quad \ldots\ldots\ldots (27)$$

Equation (27) represents the general form. It may be further simplified in the case of driven sheet-pile bulkheads in quasi-level ground since, for $y = 0$, the active pressure is balanced, and the first term may be neglected. The specific passive resistance then becomes,

$$e_p = \frac{n\,\sqrt{y}\,(z-h)^{0.5m}}{2\,\sqrt{b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The slope then becomes,

$$\frac{de_p}{dz} = \frac{nm\,\sqrt{y}}{4\,\sqrt{b}}\,(z-h)^{0.5\,m-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

and the curvature,

$$\frac{d^2e_p}{dz^2} = \frac{(m-2)\,nm\,\sqrt{y}}{8\,\sqrt{b}}\,(z-h)^{0.5\,m-2} \quad \ldots\ldots\ldots\ldots (30)$$

The vertical, specific pressure due to the weight of the material alone at a depth, $(z - h)$, below the surface, is $\gamma\,(z - h)$, and the ratio, $e_p : \lambda\,(z - h)$, is:

$$\frac{e_p}{\gamma\,(z-h)} = \xi = \xi_0 - \frac{23\sqrt[10]{y}}{\gamma} + \frac{n\sqrt{y}}{2\,\gamma\sqrt{b}}\,(z-h)^{0.5\,m-1} \quad \ldots\ldots(31)$$

which shows that ξ decreases with the depth.

According to the present theory,

$$\xi = \tan^2\left(45 + \frac{\phi}{2}\right) \quad \ldots\ldots\ldots\ldots(32)$$

in which, ϕ is the angle of internal friction offering resistance to sliding along the surface of rupture. Solving for ϕ:

$$\phi = 2 \text{ arc tan } \sqrt{\xi} - \frac{\pi}{2} \quad \ldots\ldots\ldots\ldots(33)$$

The influence of wall friction is neglected. The feature of primary interest is the decrease of ϕ with the depth similar to δ and the increase with y. This checks qualitatively at least the results of experiments on active pressure by Jacob Feld,[10] Assoc. M. Am. Soc. C. E., and Professor Terzaghi.[11]

If, as stated previously, wall friction is neglected and $\gamma\,(z - h)$ and e_p are assumed to be principal stresses (that is, horizontal and vertical shear are zero); if $(z - h)$ and η are the co-ordinates of a point of the surface of rupture; and, if ϵ is the angle between a differential length of the surface of rupture and the vertical:

$$\frac{d\eta}{dz} = -\tan\epsilon = -\sqrt{\xi} = -\sqrt{\xi_0\,\frac{-23\sqrt[10]{y}}{\gamma} + \frac{n\sqrt{y}}{2\,\gamma\sqrt{b}}\,(z-h)^{0.5\,m-1}}$$

$$= -\sqrt{\frac{n\sqrt{y}}{2\sqrt{b}}}\,(z-h)^{0.25\,(m-2)}\left\{1 + \frac{\dfrac{\xi_0 - 23\sqrt[10]{y}}{\gamma}}{\dfrac{n\sqrt{y}}{2\,\gamma\sqrt{b}}}\,(z-h)^{1-0.5\,m}\right\}^{\frac{1}{2}}$$

$$= -\,c_1\,(z-h)^{0.25(m-2)}\left\{1 + c_2\,(z-h)^{1-0.5m}\right\}^{\frac{1}{2}} \quad \ldots\ldots\ldots (34)$$

The term in the bracket equals $1 + \frac{1}{2}\,c_2\,(z - h)^{1-0.5m} - \frac{1}{8}\,c_2^2\,(z - h)^{2-m} + \ldots$. For $y = 0.10$: Equation (34) yields $c_2 = 0.0407$; and $c_2^2 = 0.00165$; and for $y = 0.30$: $c_2 = 0.0216$; and $c_2^2 = 0.000466$. For $y = 0.10$ and $(z-h) = 1$, the term in the bracket yields $1 + 0.02035 - 0.000207 + \ldots$. For $y = 0.30$ and $(z - h) = 1 : 1 + 0.0108 - 0.000058 + \ldots$; while for $y = 0.10$ and $(z - h) = 10 : 1 + 0.110 - 0.006$; and for $y = 0.30$ and $(z - h) = 10$: $1 + 0.058 - 0.0017 \ldots$.

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[10] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

[11] *Engineering*, June 13, 1930, p. 754 (Fig. 20).

Neglecting the third member:

$$\frac{d\eta}{dz} = -\left\{ c_1 \, (z - h)^{0.25\,(m-2)} + \frac{c_1\, c_2}{2} \, (z - h)^{0.25\,(2-m)} \right\} \ldots\ldots (35)$$

Integrating, Equation (35) becomes (for $z = H$ and $\eta = 0$):

$$\eta = -\left\{ \frac{4\, c_1}{m + 2} \, (z - h)^{0.25\,(m+2)} + \frac{2\, c_1\, c_2}{6 - m} \, (z - h)^{0.25\,(6-m)} \right\} + C \ldots (36)$$

in which,

$$C = + \frac{4\, c_1}{m + 2} \, (H - h)^{0.25\,(m+2)} + \frac{2\, c_1\, c_2}{6 - m} \, (H - h)^{0.25\,(6-m)} \ldots (37)$$

and the equation for η is:

$$\eta = \frac{4\, c_1}{m + 2} \left\{ (H - h)^{0.25\,(m+2)} - (z - h)^{0.25\,(m+2)} \right\}$$

$$+ \frac{2\, c_1\, c_2}{6 - m} \left\{ (H - h)^{0.25\,(6-m)} - (z - h)^{0.25\,(6-m)} \right\} \ldots\ldots\ldots\ldots (38)$$

in which,

$$c_1 = \sqrt{\frac{n}{2\, \gamma \sqrt{\frac{b}{y}}}}, \text{ and } c_2 = 2\, \frac{(\gamma\, \xi_0 - 23\, \sqrt[10]{y}}{n} \sqrt{\frac{b}{y}}$$

The shape of this surface of rupture indicates that the premise on which the derivation of its formula (Equation (38)) is based—that e_p and $\gamma\,(z - h)$, are principal stresses—is erroneous. In other words, for a rough wall, shear is active along horizontal and vertical planes.

For a frictionless wall (which, of course, never occurs in practice), the equation for the plane of rupture should read:

$$\eta = A + B\,(z - h) \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which, A, from observation, is $\dfrac{H - h}{\tan\left(45 - \dfrac{\phi}{2}\right)}$, and,

$$B = -\frac{A}{H - h} = -\frac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} \ldots\ldots\ldots\ldots (40)$$

Equation (39) then reads:

$$\eta = \frac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} \left\{ (H - h) - (z - h) \right\} \ldots\ldots\ldots\ldots (41)$$

Comparing this with Equation (38): $\dfrac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} = 2\, c_1 \left(\dfrac{2}{m+2} + \dfrac{c_2}{6-m}\right)$; $\dfrac{m+2}{4} = 1$; and, $\dfrac{6-m}{4} = 1$. For $m = 2$, $\dfrac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} = c_1 \left(1 + \dfrac{c_2}{2}\right)$.

For $\phi = 30°$, therefore, $\dfrac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} = 1.735$, and, as before, c_1 and c_2 for corresponding values of y are:

y	c_1	c_2
0.01	1.355	0.1394
0.10	2.405	0.0407
0.20	2.865	0.024
0.30	3.170	0.0216

Plotting these values, $c_1\left(1 + \dfrac{c_2}{2}\right) = 1.735$ for $y = 0.02$ ft (see Fig. 12). On the other hand, for $y = 0.40$ ft, which would correspond to a movement

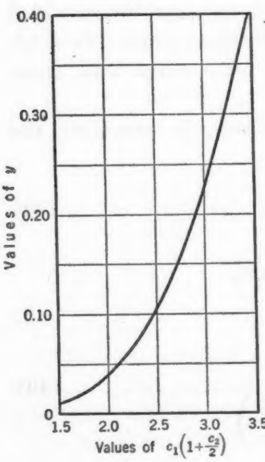


FIG. 12.

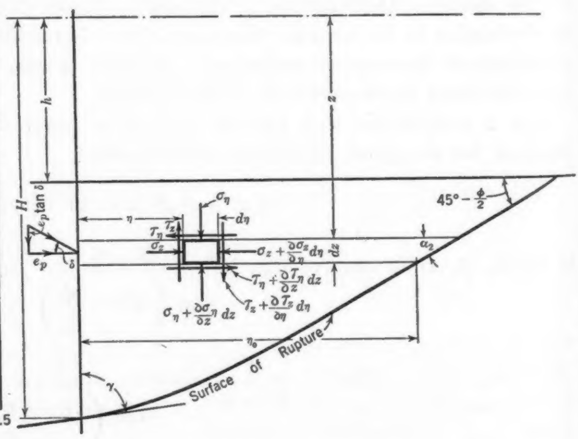


FIG. 13.

approaching the upper, practical limit, the term, $c_1\left(1 + \dfrac{c_2}{2}\right) = 3.45$, which would indicate an angle of internal friction, $\phi = 59$ degrees.

Introducing $m = 2$ in Equation (27),

$$e_p = \left\{ \left(\gamma \, \xi_o - 23 \sqrt[10]{y}\right) + \frac{n_o \sqrt{y}}{2 \sqrt{b_o}} \right\} (z - h) \dots\dots\dots (42)$$

which means that for a constant value of y the distribution of the specific, passive resistance becomes linear. The term in the bracket corresponds to $\lambda_p = \gamma \tan^2 \left(45 + \frac{\phi}{2}\right)$, and ϕ, which is now independent of z, could be determined as soon as n_o and b_o are obtained from tests.

It is evident from the foregoing that the friction between the sand and the wall, which in practice is always present to a greater or less degree, is responsible for most of the complexities in the phenomenon of passive resistance.

Possible Stress Distribution in Passive Prism for δ not Equal to Zero and Equation of Surface of Rupture

Assuming a parallelopipedon (Fig. 13), acted upon by both normal and tangential forces, the conditions of equilibrium may be expressed as follows:

$$\frac{-\partial \sigma_z}{\partial \eta} + \frac{\partial \tau_\eta}{\partial z} = 0 \dots\dots\dots (43)$$

$$\frac{-\partial \sigma_y}{\partial z} + \frac{\partial \tau_z}{\partial \eta} + \gamma = 0 \dots\dots\dots (44)$$

and,

$$\tau_\eta + \tau_z = 0 \dots\dots\dots (45)$$

For the shear stress, τ_z,

$$\tau_z \, dz = e_p \tan \delta \, dz \dots\dots\dots (46)$$

or,

$$\tau_z = e_p \tan \delta \dots\dots\dots (47)$$

in which, $\tan \delta = f_1(z)$. Then, $\frac{\partial \tau_z}{\partial \eta} = 0$; and, because $\sigma_\eta = \gamma (z - h)$:

$$\frac{\partial \sigma_\eta}{\partial z} = \gamma \dots\dots\dots (48)$$

which satisfies Equation (44).

From Equations (43) and (45):

$$\frac{\partial \sigma_z}{\partial \eta} = \frac{\partial \tau_\eta}{\partial z} = - \left\{ \frac{d e_p}{dz} \tan \delta + e_p \frac{d (\tan \delta)}{dz} \right\} = - f_2(z)$$

Integrating: $\sigma_z = - f_2(z) [\eta]_0^{(\eta_o)} + C$.

Factor $\eta = 0$ when $\sigma_z = e_p$; therefore, $C = e_p$ and,

$$\sigma_z = - f_2(z) [\eta]_0^{(\eta_o)} + e_p \dots\dots\dots (49)$$

The unit stresses then are:

$$\sigma_z = e_p - f_2(z)\,[\eta]_0^{(\eta o)} \qquad (50)$$

$$\sigma_\eta = \gamma\,(z - h) \qquad (51)$$

and,

$$\tau_\eta = -\tau_z = e_p \tan\delta = \pm\,\tau \qquad (52)$$

The principal normal stresses are:

$$\sigma_{\text{max.}} = \frac{\sigma_\eta + \sigma_z}{2} + \tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2} \qquad (53)$$

and,

$$\sigma_{\text{min.}} = \frac{\sigma_\eta + \sigma_z}{2} - \tfrac{1}{2}\sqrt{4\,\tau^2 + \sigma_\eta - (\sigma_z)^2} \qquad (54)$$

The direction of these principal stresses follows from,

$$\tan 2\alpha_1 = \frac{2\,e_p \tan\delta}{e_p - f_2(z)\,[\eta]_0^{(\eta o)} - \gamma\,(z - h)} \qquad (55)$$

The principal shear stresses are.

$$\tau_{\text{max.}} = +\tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2}$$

and,

$$\tau_{\text{min.}} = -\tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2}$$

and the direction in which they occur follows from:

$$\tan 2\alpha_2 = \frac{\gamma\,(z - h) - e_p + f_2(z)\,[\eta]_0^{(\eta o)}}{2\,e_p \tan\delta} \qquad (56)$$

From the condition that the slope of the surface of rupture and the direction of maximum shear must coincide:

$$\tan 2\alpha_2 = \tan 2\,(90 - \epsilon) - \frac{2\,\dfrac{dz}{d\eta}}{1 - \left(\dfrac{dz}{d\eta}\right)^2} \qquad (57)$$

and a differential equation of the surface of rupture is obtained as follows:

$$+\left[\left(\frac{d\eta}{dz}\right)^2 - 1\right]\left[\gamma\,(z - h) - e_p + f_2(z)\,\eta\right] - \frac{4\,d\eta}{dz}\,e_p \tan\delta = 0 \qquad (58)$$

The writer is not aware of a general solution of Equation (58). Using $\tan\delta =$ constant as a first approximation, Equation (56) gives plausible values of α_2 except near the ground surface.

The surface of rupture could be expressed satisfactorily by an hyperbolic spiral with its polar axis inclined under an angle of $45 - \dfrac{\phi}{2}$ to the horizontal and its pole in front of and above the bottom of the wall. The normal dis-

tance of the polar axis from the foot of the wall would then be a function of the wall friction and, for a frictionless wall ($\tan \delta = 0$), this distance would be zero.

Dr. Alfred Streck derives[12] an equation for E_p which is based on the curved shape of a surface of rupture in its lower part and on friction between the sand and the wall. The upper part of the surface of rupture, based on observation, is assumed to be straight, and the dividing point between the straight and the curved part is found by drawing a straight line at an angle of $45 - \frac{\phi}{2}$ to the horizontal, through the top of the wall. The tangent to the surface of rupture at the bottom of the wall is determined by means of Winkler's stress ellipse; that is, by the relation,

$$\sin (2 \beta + \delta) = \frac{\sin \delta}{\sin \phi} \quad \text{.....................(59)}$$

in which, β = the angle between the wall and the major axis of the ellipse; δ, the angle of friction between the sand and the wall; and ϕ, the angle of internal friction of sand.

Dr. Streck shows the result of his research work in a diagram from which may be obtained the effectiveness factor, μ, if ϕ and δ are known. He also shows values of μ for $\phi = 35°$ and δ = variable, based on the assumption of a plane surface of rupture to the effect that such an assumption is not on the side of safety, particularly for $\delta > 15$ degrees.

Up to $\delta = 15°$ the maximum error is about 12%; that is, $\mu = 5.50$ for a curved surface and 6.20 for a plane surface. The passive resistance for a wall of a height, h, then is: $E_p = \frac{1}{2} \gamma \mu h^2$.

The Design of Bulkheads, a Statically Indeterminate Problem

Despite the refinements suggested by Brennecke-Lohmeyer, as previously described, the present theory of bulkhead design is defective in assuming passive resistance to be static. In other words, it is based on the assumption that soil with a certain characteristic offers a fixed passive resistance, irrespective of the movement of the wall.

The fallacy of this premise becomes quite evident if two sheet-pile walls are compared, having equal lengths and widths above and below ground, the section moduli of which are considerably different and are driven in exactly the same soil.

According to the present method of design, the elastic lines of the two walls will differ due to the different section moduli only, and the passive resistance will remain constant. The only adjustment required to satisfy this condition is in changing the polar distance of the D-polygon by the ratio of the two section moduli. However, from the foregoing, it is evident that passive resistance is a function of both horizontal movement and depth and that it varies much more with the former than with the latter.

[12] "Beitrag zur Frage des Erdwiderstandes," von Alfred Streck, *Der Bauingenieur*, 1926, Hefte 1 und 2.

The differential equation of the elastic line below the original ground surface for a triangular active load is:

$$EI \frac{d^4y}{dz^4} = \lambda_a (z - h) - \frac{n \sqrt{y}}{2 \sqrt{b}} (z - h)^{0.5m} = a - p \ldots\ldots (60)$$

the solution of which is very involved indeed. The only boundary conditions known are $y = 0$ and $\frac{dy}{dz} = 0$ for $z = \infty$. The elastic line thus is comparable with a wave which is gradually diminishing as the depth below the ground increases.

Statically, a sheet-pile bulkhead is a beam, simply supported at or near the upper end and on an infinite number of elastic supports below the ground surface.

If y_0 denotes the yield of the soil at a depth, $z - h$, due to a unit load, then the yield due to a load, p, is $y_0 p$, and if the lengthening of the tie-rod due to a unit force is C_0, then the lengthening due to a force, A_0, is $C_0 A_0$. The yield, y_0, follows from $e_p = 1$; namely,

$$y_0 = \frac{4\, b}{n^2 (z - h)^m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

and,

$$C_0 = \frac{L}{E A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

in which, L = the length of the tie-rod; and, A, the cross-sectional area of the rod. As these forces grow from zero to their full value during deformation, the general equation of the total work of deformation, or the stored-up energy per linear width of wall is:

$$A = \tfrac{1}{2} C_0 A_0^2 + \tfrac{1}{2} \int_h^{\infty} y_0\, p^2\, dz + \int_0^{\infty} \tfrac{1}{2} \frac{M^2}{EI} dz \ldots\ldots\ldots (63)$$

in which,

$$M = A_0 z - \int_{x=0}^{x=h} a\, (z - x)\, dx + \int_{x=h}^{x=z} p\, (z - x)\, dx \ldots\ldots (64)$$

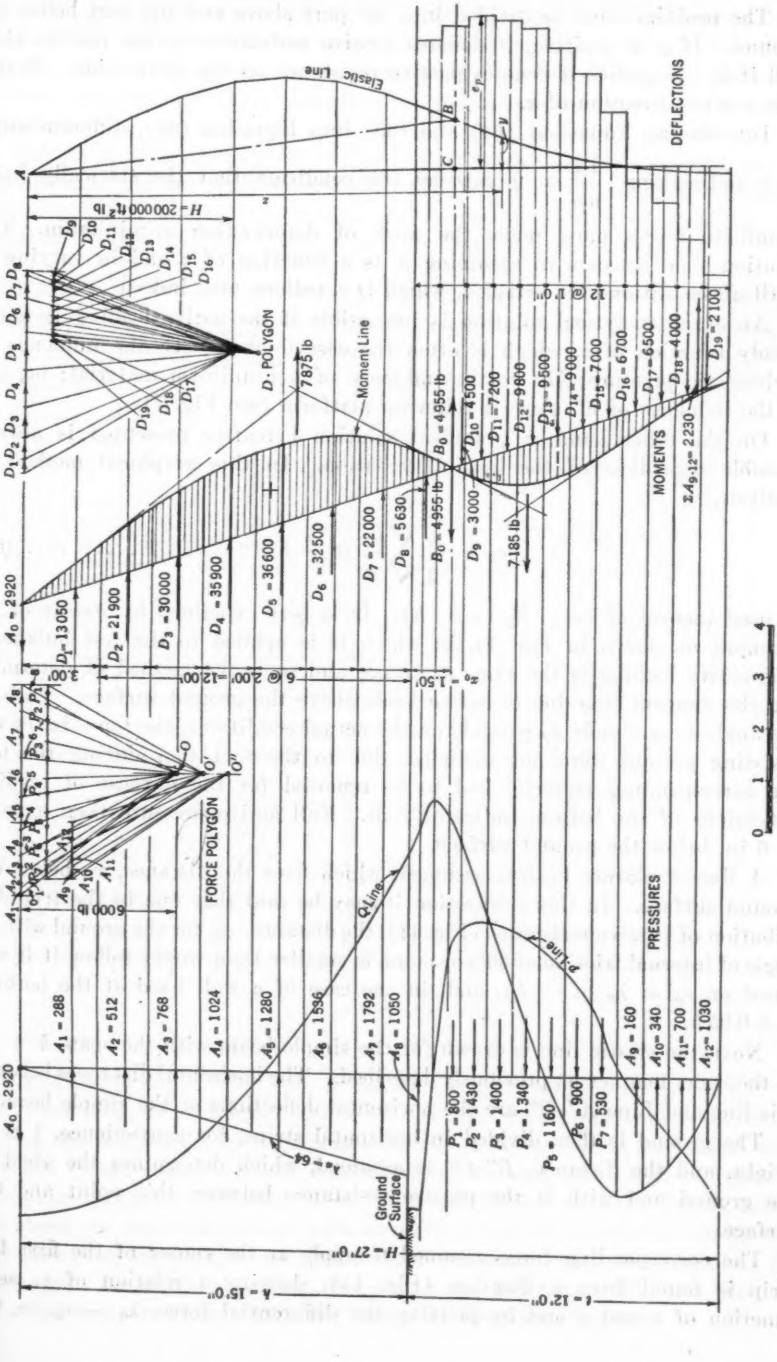
and a = the active pressure. As the amplitude of the wave, representing the elastic line, diminishes rapidly below the ground surface, the boundary conditions, $y = 0$ and $\frac{dy}{dz} = 0$, will be satisfied closely enough for practical purposes at a depth, $z = H$, and Equation (63) becomes finite.

For equilibrium:

$$A_0 - \int_0^h a\, dz + \int_h^H p\, dz = 0 \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

and,

$$- \int_0^h a\, z\, dz + \int_h^H p\, z\, dz = 0 \ldots\ldots\ldots\ldots\ldots\ldots (66)$$



DEFLECTIONS
Elastic Line
MOMENTS
Moment Line
D - POLYGON
H = 200000 ft² lb
7875 lb
B_0 = 4955 lb
7185 lb
PRESSURES
FORCE POLYGON
Q Line
Ground Surface
H = 27.0"
λ = 15.0"
t = 12.0"
Vertical Scale in Inches

FIG. 14.

The problem must be divided into the part above and the part below the ground. If p is positive, it denotes passive resistance on the passive side, and if it is negative it denotes passive resistance on the active side. Factor a is a given function of z.

Introducing Equations (64) and (65) into Equation (63), differentiating by p, and making $\frac{dA}{dp} = 0$, satisfies the condition that the statically indeterminate forces must make the work of deformation a minimum. The solution then consists in assuming p as a function of z and in varying it until all conditions are satisfied, which is a tedious task indeed.

An exact analytical solution is impossible if the active loading is not a steady function of z, which is often the case due: (a) To the influence of hydrostatic pressure; (b) to the influence of non-uniform material; or, (c) to the influence of fill above a relieving platform (see Fig. 7).

On the other hand, a graphical solution hereafter presented is always possible regardless of the load distribution. In this graphical method of analysis,

$$e_p = \frac{n \sqrt{y}}{2 \sqrt{b}} (z - h)^{0.5m} \qquad \text{(67)}$$

is used instead of $e_p = \lambda_p (z - h)$. It is best explained by means of an example, as shown in Fig. 14, in which it is applied to the test bulkhead. The active loading is the same as before and so is the method of determining the moment line due to active loads above the ground surface. To base the analysis as closely as possible on the actual conditions, the top 6 in. of the resisting ground were not included, due to the fact that during the test the corresponding material had to be removed for the purpose of reading deflections of the bottom indicator rods. Full active pressure then extends to 6 in. below the ground surface.

A line of closure is then assumed, which fixes the distance, x_o, below the ground surface. In this connection it may be said that due to the true distribution of passive resistance (Fig. 11), the distance, x_o, for the ground with an angle of internal friction of 30°, or more, is smaller than would follow if it was based on $e_p = \lambda_p (z - h)$, and, in the case of a wall fixed at the bottom, $x_o \leqq 0.05\,h$.

Next, the elastic line is drawn for the simple beam with the span, $h + x_o$, in the same manner as previously described. The horizontal distances between this line and Line A''–B'' are the horizontal deflections of the simple beam.

The ground is then divided in horizontal strips, for convenience, 1 ft in height, and the distance, B''–C'' is assumed, which determines the yield of the ground and with it the passive resistance between this point and the surface.

The corresponding force assumed to apply at the center of the first 1-ft strip is found from a diagram (Fig. 15), showing a relation of e_p as a function of z and y and by plotting the differential force, $\lambda_a - e_p$, in the

force polygon, a new ray is obtained for the extension of the moment line. In turn, the increment of D thereby determined is plotted in the D-polygon, and the corresponding ray serves to extend the elastic line to the center of the next strip below. This procedure is repeated until the moment diagram is closed.

Two or three trials may be required; that is, two or three assumptions of the distance, $B''-C''$, may have to be made, to close the moment diagram. However, an erroneous assumption will promptly be detected without the necessity of carrying the moment or the elastic line down very far.

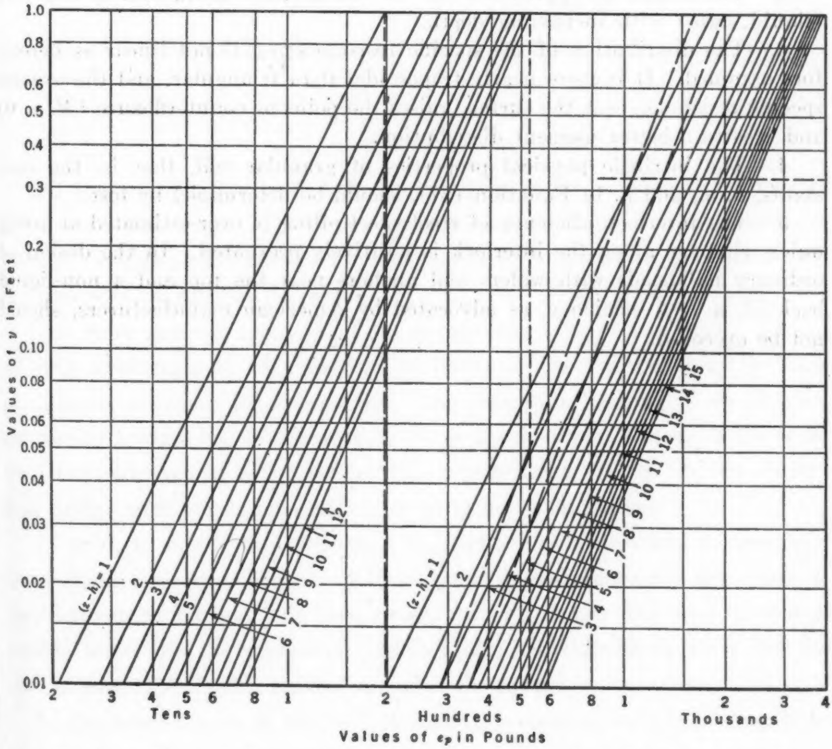


FIG. 15.—RELATION BETWEEN e_p AND y FOR CORRESPONDING VALUES OF $z - h$

As to the example (Fig. 13), it was necessary to apply a factor of 2.50 to the values of e_p to establish equilibrium. If the writer was correct in interpreting the test results of Professor Franzius as being due to a width of wall of 6.6 ft (2 m), rather than of unit width, this would mean that the passive resistance, e_p, of the hydraulic fill in the Outer Harbor at Long Beach, which was about one year old, was as much as 400% of the fill as used in the laboratory tests; but should the test results refer to a unit width, the hydraulic fill would then be about twice as effective as the laboratory fill, and the latter would compare closely with the former without wall friction.

It is interesting to note that the top reaction, A_0, as determined by this new method of analysis, checks the one obtained from extensometer readings on the tie-rods within 1 per cent.

Conclusions

From the foregoing studies, the following conclusions can be drawn:

1.—The angle of internal friction, ϕ, of granular soil resisting the pressure of a rough wall, and the angle of friction, δ, between the wall and the soil decrease with the depth and increase with the path of the wall.

2.—The angle, ϕ, approaches 90° at the surface (solid state) and zero (liquid state) with increasing depth.

3.—The distribution of the specific resistance, e_p, is not linear as heretofore assumed. It is more nearly trapezoidal than triangular, and the greater specific resistance near the surface raises the point of counterflexure ($M = 0$) and creates a better moment distribution.

4.—The intrinsic physical properties of granular soil, that is, the constants, b, m, and n, in Equation (27), should be determined by test.

5.—The interlock efficiency of steel sheet-piling is over-estimated at 100% unless slippage along the interlock is positively prevented. In the design of ordinary bulkheads with walers and anchors near the top and a non-liquid back-fill, a 75% efficiency, as advocated by American manufacturers, should not be exceeded.

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PAPERS

A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES

By D. B. Steinman,[1] M. Am. Soc. C. E.

Synopsis

An improved and extended Deflection Theory for suspension bridges is presented in this paper. The theory is generalized to include structures of any number of spans, continuous or non-continuous, symmetrical or unsymmetrical, and with or without tie cables.

The more general adoption of the continuous type of suspension bridge, offering advantages of economy and rigidity, has been retarded by the lack of an accurate theory for its analysis. The Deflection Theory for simple-span suspension bridges has been available to the Engineering Profession for more than forty-five years; but, thus far, the corresponding theory for the suspension bridge with continuous stiffening truss has been lacking.

In order to supply this deficiency, the writer has undertaken to develop a Generalized Deflection Theory, applicable to both continuous and non-continuous, three-span, and multiple-span structures. By simply dropping the recognizable terms due to continuity, the formulas are reduced to those for the simpler case of stiffening trusses hinged at the towers.

In the development of the analysis herein presented, maximum simplicity of formulas and ease of practical application have been governing considerations. Incidentally, new simplifications are here developed and introduced in the working formulas hitherto published for the two-hinged type.

To show the practical workability of the Generalized Theory, the paper includes a numerical example of the application of the formulas to the calculation of the stresses and deflections in a continuous suspension bridge of 800-ft main span.

Note.—Discussion on this paper will be closed in August, 1934, *Proceedings*.

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1.—INTRODUCTION

The common or approximate theory for the stress analysis of stiffened suspension bridges is known as the Elastic Theory. The values of the bending moments and shears yielded by this method are too high; they satisfy safety but not economy. The error increases with the flexibility of the structure, the span length, and the ratio of dead load to live load.

A more exact method of analysis, which takes into account the deformed configuration of the structure, is known as the Deflection Theory. It yields lower stresses and a consequent saving (ranging normally from 20 to 65%) of metal in the stiffening truss.

The Deflection Theory, or "More Exact Theory," as applied to non-continuous suspension bridges, was originated by J. Melan and was first published by him in 1888 in his classic work "Theorie der eisernen Bogenbrücken und der Hängebrücken." (It was republished in 1906 in the Third Edition, which was translated by the writer[2] in 1909. In the Fourth Edition (1925), the Deflection Theory appears again, in amplified form.) The working formulas were amplified by L. S. Moisseiff, M. Am. Soc. C. E. (and independently by the writer), in 1909, to cover the case of suspended side spans, and the theory has since been published in its extended form by F. E. Turneaure, Hon. M. Am. Soc. C. E.,[3] and by the writer.[4]

The Deflection Theory, as hitherto developed, is not directly applicable to suspension bridges with continuous stiffening trusses. A number of such structures have been built, including the Rondout Bridge, at Kingston, N. Y., with a 705-ft main span (1922), the General U. S. Grant Bridge over the Ohio River, at Portsmouth, Ohio, with a 700-ft main span (1927), the Sixth, Seventh, and Ninth Street Bridges over the Allegheny River, at Pittsburgh, Pa., and several bridges in Europe; but the designers have been handicapped by the lack of an accurate method of analysis. Without correct proportioning, the full economy of this type of suspension bridge can not be secured.

Continuous stiffening trusses offer greater efficiency than the conventional two-hinged type. This increased efficiency may be in the form of superior economy, or superior rigidity, or both, depending on the proportions adopted. The hingeless type offers incidental advantages in greater efficiency of the continuous lateral truss, in improved and simplified supporting details at the towers, and in reduced variation between minimum and maximum sections. For suspension bridges of less than 1 000-ft main span, the continuous stiffening truss may well be considered the superior type.

In past designs of continuous suspension bridges, the deflection corrections have been either neglected or conservatively approximated. Had the theory and formulas presented in this paper been then available, the stresses calculated could have been reduced approximately 30% for the Rondout Bridge,

[2] "Theory of Arches and Suspension Bridges," by D. B. Steinman, M. Am. Soc. C. E., M. C. Clark Pub. Co., Chic., 1913.

[3] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Hon. M. Am. Soc. C. E., Ninth Edition, Pt. 2, N. Y., John Wiley & Sons, 1911, and succeeding editions.

[4] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, M. Am. Soc. C. E., Second Edition, N. Y., John Wiley & Sons, 1929.

and 18% for the Portsmouth Bridge. For the more modern stiffening truss design of 800-ft main span, used as a practical example in this paper, the theory here developed yields approximately 45% reduction in stresses and sections. With the Deflection Theory for such continuous structures available, more scientifically proportioned designs can be made, and the economic utilization of this bridge type will be facilitated.

Multiple-span suspension bridges also offer possibilities that have not been fully explored. The generalization of the Deflection Theory to permit the accurate analysis of multiple-span structures should facilitate their further investigation.

2.—NOTATION

The general notation to be used is shown in Fig. 1. The subscripts (as in I_1, l_1, f_1, x_1, y_1, η_1) are added to distinguish corresponding side-span magnitudes. The initial loading, w, for which the stiffening truss is unstressed

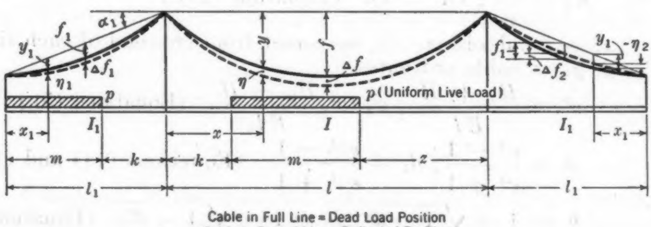


FIG. 1.—NOTATION DIAGRAM

and undeflected, will be called the "dead load." The load, p, applied subsequently, which produces stress and deflection, will be termed the "live load."

The notation used in the formulas (with the numbers of pertinent equations, for convenience of reference) is as follows:

H_w = horizontal tension in cable due to dead load $(w + g)$.
H = horizontal tension in cable due to live load, p, and temperature, t. (Equations (24), (27), (37), and (47).)
M_0 = simple beam bending moment due to live load, p.
M = total resultant bending moment at any section, x, of stiffening truss. (Equations (4), (6), (9), and (29).)
T = bending moment at any section, x, due to continuity. (Equations (3).)
$T_1, T_2, \ldots$ = bending moments in truss at towers. (Equations (19) and Article 11.)
V = total shear at any section, x, of stiffening truss. (Equation (10).)
η, η_1 = deflection of truss at any section, x, x_1. (Equation (7).)
I, I_1 = moments of inertia of stiffening truss.
C_1, C_2 = pair of integration constants. (Equations (12), (13), and (30).)
E = elastic modulus of truss; E_c = elastic modulus of cable.
L_s, L_t = cable-length functions, defined by Equations (21).
p_x = intensity of live load at any section, x.
p_s = live load carried by the cable. (Equation (11).)

p_t = live load carried by the truss. (Equation (11).)
t = temperature change.
ω = coefficient of expansion.
A_c = cable section at any point; A_0 = cable section at mid-span.
α = slope of cable chord in any span. (Equations (21).)
s = length of cable arc. (Equations (21).)
l, l_1 = span lengths.
f, f_1 = cable sags.
$n = \frac{f}{l}$.
$n_1 = \frac{f_1}{l_1}$.
$\Delta f, \Delta f_1$ = mid-span deflections. (Equations (33).)
$r = \frac{l^2}{8f}$; $r_1 = \frac{l^2_1}{8f_1}$. (Equations (8).)
$K_1 = \frac{r}{r_1}$; $(K = 1)$. (Equation (23).)
x, x_1 = abscissas. (x_1 measured from free end of each side span.)
y, y_1 = cable ordinates.
$c^2 = \frac{H_w + H}{EI}$; $c^2_1 = \frac{H_w + H}{EI_1}$. (Equations (5).)
$d = \frac{e^{cl} - 1}{e^{cl} + 1}$; $d_1 = \frac{e^{c_1 l_1} - 1}{e^{c_1 l_1} + 1}$. (Equations (14) and (15).)
$h = 1 - \sqrt{1 - d^2}$; $h_1 = 1 - \sqrt{1 - d^2_1}$. (Equations (31).)
$a = cd + \frac{c}{d} - \frac{2}{l}$; $a_1 = c_1 d_1 + \frac{c_1}{d_1} - \frac{2}{l_1}$. (Equations (41*a*).)
$b = -cd + \frac{c}{d} - \frac{2}{l}$. (Equations (41*b*).)
$L_c = l - \frac{2d}{c}$; $L_{c1} = l_1 - \frac{2d_1}{c_1}$. (Equations (26).)
N = denominator of T-formula. (Equation (19*a*).)
F = term in T-formula (Equation (19*a*)) $= \Sigma' (KL_c)$. (Equation (19*c*).)
A = load function used in H-formulas; tabulated in Article 9.
B_1, B_2 = load functions in T-formulas; tabulated in Article 9.
G_1, G_2, G_3 = load functions in formulas for C_1 and C_2; tabulated in Article 9.
W_1, W_2 = work quantities. (Article 7.)
Σ = summation of similar terms for all spans.
Σ' = weighted summation for all spans with one-half weight for end spans.
S, S_1, S_2 = load functions, defined in Equations (28).
k, m, z = span segments. (Article 9.)
R_L, R_R = simple-beam end reactions at $x = 0$ and $x = l$. (Equations (17), (20*a*), and (38).)
$U_1, U_2 \ldots$ = slope-change functions at towers (Equations (18) and (38).)
Y_L, Y_R = functions of R and G. (Equations (20).)
Y'_L, Y'_R = functions of R, G, and H. (Equations (42).)

3.—Fundamental Assumptions

The Deflection Theory for the analysis of continuous and multiple-span suspension bridges is based on the same assumptions as the corresponding theory for two-hinged suspension bridges, namely: (1) The initial curve of the cable is a parabola (in practice, the greatest ordinate deviation from a true parabola is seldom as great as $\frac{1}{2}\%$); and, (2) the initial dead load, w, is carried by the cable (producing the initial horizontal tension, H_w), without causing stress in the stiffening truss.

Unlike the Elastic Theory, the Deflection Theory does not assume that the ordinates, y, of the cable curve remain unaltered on application of the loading. In other words, the alteration of the lever arms of the cable forces is taken into account. This change in cable ordinates or lever arms makes the initial cable tension, H_w, significant.

The theory that follows is applicable to either continuous or non-continuous suspension bridges, with or without suspenders in the side spans. For simplicity of notation, a symmetrical three-span suspension bridge is first assumed. All the formulas, however, except where noted, are also applicable to other cases. The few necessary modifications for complete generality, to cover unsymmetrical and multiple-span structures, are presented in Article 11.

4.—Fundamental Equations

If the deflections of the span are neglected, the bending moment at any point of a two-hinged suspension span is given by the basic formula of the Elastic Theory:

$$M = M_0 - H\,y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, M_0 denotes the simple-span bending moment and $H\,y$ represents the relieving moment due to live-load cable tension, H, acting through the suspender forces.

In consequence of the deflection, η, the bending moments are relieved by an additional amount, $(H_w + H)\eta$, and the expression for M becomes:

$$M = (M_0 - H\,y) - (H_w + H)\eta \ldots\ldots\ldots\ldots\ldots (2)$$

If the spans are continuous at the towers, let T_1 and T_2 denote the total resultant bending moments (including correction for deflections) at the left and right towers, respectively. Then the contribution of continuity to the bending moment at any section will be:

For the Main Span:

$$T = \frac{l - x}{l}\,T_1 + \frac{x}{l}\,T_2 \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

For the Side Spans:

$$T = \frac{x_1}{l_1}\,T_{1,2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

With this contribution, T, added to the expression of Equation (2), the total resultant bending moment at any section of a continuous suspension bridge will be:

$$M = (M_o - H\,y) - (H_w + H)\,\eta + T \qquad (4)$$

This is the basic equation of the Deflection Theory for continuous suspension bridges. It is also the basic equation of the Generalized Deflection Theory, applicable to simple, continuous, and multiple-span suspension bridges.

By introducing the symbol, c^2,

For the Main Span:

$$c^2 = \frac{H_w + H}{E\,I} \qquad (5a)$$

For the Side Spans:

$$c^2_1 = \frac{H_w + H}{E\,I_1} \qquad (5b)$$

Equation (4) becomes:

$$M = (M_o - H\,y) - (E\,I\,c^2)\,\eta + T \qquad (6)$$

Neglecting the elongation of the suspenders, the truss at any point will have the same deflection, η, as the cable at that point.

By the common theory of flexure applied to the truss, $\frac{d^2\eta}{dx^2} = -\frac{M}{E\,I}$.

Substituting Equation (6),

$$\frac{d^2\eta}{dx^2} = c^2\eta - \frac{1}{E\,I}\,(M_o - H\,y + T)$$

The solution of this differential equation yields the general formula for deflections, or the equation of the deflection curve:

$$\eta = \frac{1}{E\,I\,c^2}\left[(C_1\,e^{cx} + C_2\,e^{-cx}) - \frac{1}{c^2}\left(p_x - \frac{H}{r}\right) + (M_o + T - H\,y)\right] \qquad (7)$$

in which, p_x is the live load per unit length at the section, x, and r is the parameter of the cable parabola, given by:

$$r = \frac{l^2}{8\,f} \qquad (8a)$$

and,

$$r_1 = \frac{l^2_1}{8\,f_1} \qquad (8b)$$

Substituting Equation (7) in Equation (6), the general formula for M, or the equation of the M-curve, is:

$$M = -\left[(C_1\,e^{cx} + C_2\,e^{-cx}) - \frac{1}{c^2}\left(p_x - \frac{H}{r}\right)\right] \qquad (9)$$

From Equation (9) it is observed that the bending moment, M, is not simply proportional to the load, p, that produces it. Equation (4) also shows that the value of M is affected by the dead load stress, H_w, in the cable before the application of the live load. In the Deflection Theory, influence lines (in the ordinary sense) cannot be used. Stresses producible by combinations of loadings cannot be found by adding algebraically the respective stresses producible by the component loadings.

The general formula for shears V, or the equation of the V-curve, is obtained by differentiating Equation (9), which gives:

$$V = \frac{dM}{dx} = -c\left[C_1 e^{cx} - C_2 e^{-cx}\right] \ldots\ldots\ldots\ldots (10)$$

Differentiating Equation (10), an expression is obtained for the live load per unit length that is actually carried by the stiffening truss at any point, x, as follows:

$$p_t = (p_x - p_s) = -\frac{d^2 M}{dx^2} = -\frac{dV}{dx} = c^2\left[C_1 e^{cx} + C_2 e^{-cx}\right] \ldots (11)$$

Equation (11) shows that the suspender loading, p_s, is no longer constant, as in the Elastic Theory, but becomes a variable in the Deflection Theory.

Equations (4), (6), (7), (9), (10), and (11) are applicable to points in the side spans as well as to those in the main span; all that is necessary is to write the subscript symbols, x_1, y_1, l_1, f_1, r_1, I_1, c_1, and η_1, instead of the corresponding main-span quantities. For non-continuous suspension bridges, simply write zero for T wherever it occurs in this Article. The formulas of this Article are directly applicable, without change, to multiple-span suspension bridges as well as to unsymmetrical structures.

5.—Evaluation of the Integration Constants

The constants of integration, C_1 and C_2, appearing in Equations (7), (9), (10), and (11), as well as later in the basic equations for T_1, T_2, and H, must be determined for each different condition of loading. For each span segment having a constant value of p and of I, there is a pair of values for C_1 and C_2.

In the treatment that follows, it will be assumed (as is usually done for the sake of simplicity), that the moment of inertia, I (or I_1), is constant throughout the length of any span under consideration, although it may have different respective values for the three different spans. The error of ignoring the variation of I within a span is found to be practically negligible and on the side of safety.

The procedure for writing working formulas for C_1 and C_2 for the various conditions of loading, is as follows.

One Loading Segment.—For the case of the main span fully loaded with a uniform applied load, p, and assuming constant moment of inertia, I, the quantities, C_1 and C_2, are obtained from the two known conditions that, for $x = 0$ and $x = l$, $\eta = 0$ in Equation (7), or $M = T_1$ and T_2, respectively, in Equation (9). Substitute these values and solve the resulting two independent equations for C_1 and C_2.

Two Loading Segments.—For the case of a partial loading of the main span with a uniform load, p, per unit length extending a distance from either end of the span, the constants, C_1 and C_2, for the left segment, and the constants, C'_1 and C'_2, for the right segment, are obtained from the four known conditions that moment and shear at the right end of the left segment must be equal, respectively, to those at the left end of the right segment, and that $\eta = 0$ (or $M = T_{1,2}$) at each end of the span. Substitute these four relations in Equations (7), (9), and (10), and solve the resulting four equations for the two pairs of integration constants.

Three Loading Segments.—If the main span is divided into three segments, $k + m + z = l$, having different uniform loads, or alternately loaded and unloaded, the three corresponding pairs of integration constants are obtained from the six known conditions that M and V at the right end of the first segment must be equal, respectively to M and V at the left end of the second segment; that the same two equalities hold at the junction of the second and third segments; and that $\eta = 0$ (or $M = T_{1,2}$) at each end of the span. Substitute these six relations in Equations (7), (9), and (10), and solve the resulting six independent equations for the three pairs of integration constants.

By the foregoing procedure, the following working formulas are obtained for the pair of integration constants in any loaded or unloaded segment of any span:

In the Main Span:

$$C_1 = -\frac{(1-d)}{4}\left[(T_1 + T_2) - \frac{1}{d}(T_1 - T_2) + \frac{2}{c^2}\left(\frac{H}{r} - G_1\right)\right] \ldots (12a)$$

$$C_2 = -\frac{(1+d)}{4}\left[(T_1 + T_2) + \frac{1}{d}(T_1 - T_2) + \frac{2}{c^2}\left(\frac{H}{r} - G_2\right)\right] \ldots (12b)$$

and,

$$C_1 + C_2 = -T_1 - \frac{1}{c^2}\left(\frac{H}{r} - G_3\right) \ldots\ldots\ldots\ldots\ldots (12c)$$

In the Side Spans:

$$C_1 = -\frac{(1-d_1)}{4}\left[\frac{(1+d_1)}{d_1}T_{1,2} + \frac{2}{c^2_1}\left(\frac{H}{r_1} - G_1\right)\right] \ldots\ldots (13a)$$

$$C_2 = +\frac{(1+d_1)}{4}\left[\frac{(1-d_1)}{d_1}T_{1,2} - \frac{2}{c^2_1}\left(\frac{H}{r_1} - G_2\right)\right] \ldots\ldots (13b)$$

and

$$C_1 + C_2 = -\frac{1}{c^2_1}\left(\frac{H}{r_1} - G_3\right) \ldots\ldots\ldots\ldots\ldots (13c)$$

Equations (13) for the side spans may be written from the corresponding Equations (12) for the main span by simply substituting the end moments, 0 and $T_{1,2}$, of the side spans for the end moments, T_1 and T_2, respectively, of the main span, and writing c_1, d_1, and r_1 instead of c, d, and r.

In Equations (12) and (13), the quantities, G_1, G_2, and G_3, are functions of the loading. The expressions for G_1 and G_3 are tabulated for the different loading cases in Article 9. The expressions for G_2 are not tabulated, because they may easily be written, if desired, from the parallel expressions for G_1 by simply substituting $-d$ for d throughout (in the same manner as the C_2 formulas may be written from the parallel formulas for C_1). For any span fully unloaded, G_1, G_2, and G_3 are zero. For any span fully loaded, $G_1 = G_2 = G_3 = p$.

A general formula for G_1, G_2, G_3, in any segment, m, of any span having any number of segments with any uniform load (or zero loading) in any segment, may be written easily, but is here omitted.

A new abbreviation is introduced in Equations (12) and (13):

$$d = \frac{(e^{cl} - 1)}{(e^{cl} + 1)} \quad \text{.......................(14a)}$$

and,

$$d_1 = \frac{(e^{c_1 l_1} - 1)}{(e^{c_1 l_1} + 1)} \quad \text{.......................(14b)}$$

This substitution eliminates, from the working formulas of this theory, the various exponential functions involving e^{cl} or $e^{c_1 l_1}$, thereby facilitating analytical operations and numerical application. The convenience of this abbreviation in simplifying the analysis is illustrated by the following typical resultant substitutions for exponential functions that, otherwise, would occur, singly or in combination, in the formulas of this paper:

$$\frac{(e^{cl} + e^{-cl} - 2)}{(e^{cl} - e^{-cl})} = d \qquad \frac{(e^{cl} + e^{-cl} + 2)}{(e^{cl} - e^{-cl})} = \frac{1}{d}$$

$$\frac{1}{(e^{cl} + 1)} = \frac{1}{2}\left(1 - d\right) \qquad \frac{1}{(e^{cl} - 1)} = \frac{1}{2}\left(\frac{1}{d} - 1\right)$$

$$\frac{e^{cl}}{(e^{cl} + 1)} = \frac{1}{2}\left(1 + d\right) \qquad \frac{e^{cl}}{(e^{cl} - 1)} = \frac{1}{2}\left(\frac{1}{d} + 1\right)$$

$$\frac{(e^{cl} + e^{-cl})}{(e^{cl} - e^{-cl})} = \frac{1}{2}\left(d + \frac{1}{d}\right) \qquad \frac{2}{(e^{cl} - e^{-cl})} = \frac{1}{2}\left(\frac{1}{d} - d\right)$$

A table (Table 1) or graph giving the values of d (or d_1) for different values of $c\,l$ (or $c_1 l_1$) may be computed by Equations (14) or, with the aid of a table of natural hyperbolic functions, using the relation,

$$d = \frac{\sinh c\,l}{\cosh c\,l + 1} = \frac{\cosh c\,l - 1}{\sinh c\,l} = \tanh \frac{c\,l}{2} \quad \text{..........(15)}$$

The direct use of Table 1 in the numerical application of the theory takes the place of tables of exponential or hyperbolic functions, or the equivalent logarithmic operations, and eliminates the need of working with the various expressions involving the exponentials, e^{cl}, e^{-cl}, $e^{0.5cl}$, $e^{-0.5cl}$, which heretofore

have been conspicuous in Deflection Theory formulas. This substitution of d-terms for exponential expressions may easily be extended, if so desired, to the few remaining exponentials in the working formulas of this theory, thereby further condensing the formulas and completely eliminating exponential functions from the numerical solution.

TABLE 1.—VALUES OF d

cl	d	cl	d	cl	d	cl	d	cl	d	cl	d
2.5	0.84828	3.4	0.93541	4.3	0.97323	5.2	0.98903	6.1	0.99552	7.0	0.99818
2.6	0.86172	3.5	0.94138	4.4	0.97574	5.3	0.99007	6.2	0.99595	7.1	0.99835
2.7	0.87405	3.6	0.94681	4.5	0.97803	5.4	0.99101	6.3	0.99633	7.2	0.99851
2.8	0.88535	3.7	0.95175	4.6	0.98010	5.5	0.99186	6.4	0.99668	7.4	0.99878
2.9	0.89569	3.8	0.95624	4.7	0.98197	5.6	0.99263	6.5	0.99700	7.6	0.99900
3.0	0.90515	3.9	0.96032	4.8	0.98367	5.7	0.99333	6.6	0.99728	7.8	0.99918
3.1	0.91379	4.0	0.96403	4.9	0.98522	5.8	0.99396	6.7	0.99754	8.0	0.99933
3.2	0.92167	4.1	0.96740	5.0	0.98661	5.9	0.99454	6.8	0.99777	8.2	0.99945
3.3	0.92886	4.2	0.97045	5.1	0.98788	6.0	0.99505	6.9	0.99799	8.4	0.99955

In actual design it will generally be found that the values of d are between 0.96 and 0.9999, and that the values of d_1 are between 0.85 and 0.96.

It should be noted that unsymmetrically loaded spans are not reversible left to right without altering the values of the integration constants given by Equations (12) and (13), unless the origin of x is also reversed. Equations (13) for the integration constants in the side spans are correct only if x_1 is measured from the free end of the span. It should also be noted that the expressions for the integration constants, C, for any loading condition in a span, are unaffected by the loading conditions in the other spans.

Upon substituting zero for T_1 and T_2 (representing continuity), the foregoing formulas for C_1 and C_2 will reduce to the corresponding formulas for non-continuous suspension bridges. The functions, G_1, G_2, and G_3, are identical for continuous and non-continuous stiffening trusses.

The formulas of this Article for any span are not affected by the number or the proportions of the other spans. Hence, the foregoing formulas (including the G-functions) are directly applicable, without change, to multiple-span suspension bridges and to unsymmetrical structures. For the end spans of multiple-span bridges, use the side-span formulas; for any intermediate span, use the main-span formulas.

6.—EVALUATION OF T_1 AND T_2

The continuity moments, T_1 and T_2, are to be evaluated by equating deflection slopes at the towers.

Differentiating Equation (7), the general equation for the slope of the elastic curve of the stiffening truss at any point, is as follows:

$$\frac{d\eta}{dx} = \frac{1}{E I c^2}\left[c\,(C_1\,e^{cx} - C_2\,e^{-cx}) + \left(\frac{T_2 - T_1}{l} + \frac{dM_o}{dx}\right) - H\,\frac{4\,f}{l^2}\,(l - 2\,x)\right] \quad (16)$$

At the tower ends of the spans, Equation (16) yields the following values of the slope:

Main span, at $x = 0$:

$$\frac{d\eta}{dx} = \frac{1}{EI\,c^2}\left[c\,(C_1 - C_2)_{2L} + \left(\frac{T_2 - T_1}{l} + R_{2L}\right) - 4\,H\,n\right] \ldots (17a)$$

Main Span, at $x = l$:

$$\frac{d\eta}{dx} = \frac{1}{EI\,c^2}\left[c\,(C_1\,e^{cl} - C_2\,e^{-cl})_{2R} + \left(\frac{T_2 - T_1}{l} - R_{2R}\right) + 4\,H\,n\right] . (17b)$$

Left Side Span, at $x_1 = l_1$:

$$\frac{d\eta}{dx} = \frac{1}{EI\,c^2}\left[c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{1R} + \left(\frac{T_1}{l_1} - R_{1R}\right) + 4\,H\,n_1\right] \ldots (17c)$$

Right Side Span, at $x_1 = l_1$:

$$-\frac{d\eta}{dx} = -\frac{1}{EI\,c^2}\left[c_1\,(C_1\,e^{c_1 l_1} - C_2 e^{-c_1 l_1})_{3R} + \left(\frac{T_2}{l_1} - R_{3R}\right) + 4\,H\,n_1\right] \ldots (17d)$$

in which, R (with designating subscripts) denotes the simple-beam live-load reaction at the end of a span, and C_1 and C_2, with the same designating subscripts, are the pair of integration constants in the span segment at the same end of the same span.

The foregoing subscript notation will be found to be convenient later. The numeral subscript after a symbol or expression designates the number of the span, counting from left to right. The letter subscript designates the left or right end of the span (Subscript L at $x = 0$, and Subscript R at $x = l$).

Equating the two slopes at the left tower (Equations (17a) and (17c)), and the two slopes at the right tower (Equations (17b) and (17d)), and solving the resulting two simultaneous equations for T_1 and T_2:

$$(T_1 + T_2) = l_1\,(U_1 + U_2) \ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and

$$(T_1 - T_2) = \frac{l_1}{1 + 2\,\dfrac{l_1}{l}}\,(U_1 - U_2) \ldots\ldots\ldots\ldots\ldots (18b)$$

in which,

$$U_1 = (R_{1R} + R_{2L}) - c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{1R} + c\,(C_1 - C_2)_{2L} - 4\,H\,(n + n_1) \ldots\ldots\ldots\ldots\ldots\ldots (18c)$$

and,

$$U_2 = (R_{2R} + R_{3R}) - c\,(C_1\,e^{cl} - C_2\,e^{-cl})_{2R} - c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{3R} - 4\,H\,(n + n_1) \ldots\ldots\ldots\ldots\ldots\ldots (18d)$$

Substituting, in Equations (18), the general expressions for the integration constants given by Equations (12) and (13), and then solving for

$(T_1 + T_2)$ and $(T_1 - T_2)$, the following working formulas are obtained for evaluating T_1 and T_2:

$$(T_1 + T_2) = \frac{\frac{H}{r} F - \Sigma B_1}{c d + \frac{c_1}{2}\left(d_1 + \frac{1}{d_1}\right) - \frac{1}{l_1}} = \frac{\frac{H}{r} F - \Sigma B_1}{N} \quad \ldots (19a)$$

and,

$$(T_1 - T_2) = \frac{-\Sigma B_2}{\frac{c}{d} + \frac{c_1}{2}\left(d_1 + \frac{1}{d_1}\right) - \left(\frac{1}{l_1} + \frac{2}{l}\right)} = \frac{-\Sigma B_2}{(N + b)} \quad \ldots (19b)$$

in which,

$$F = 2\, r \left[4\,(n + n_1) - \left(\frac{d}{r\, c} + \frac{d_1}{r_1\, c_1}\right)\right] = \Sigma'\,(K\, L_c) \ldots\ldots (19c)$$

and in which, ΣB_1 and ΣB_2 are functions of the loading, p. These functions are related to previously derived functions, as follows: Let,

$$Y_{L, R} = R_{L, R} - \left[\frac{d}{2\,c}\,(G_1 + G_2) \mp \frac{1}{2\,c}\,(G_1 - G_2)\right]_{L, R} \ldots\ldots (20a)$$

Then the values of B_1 and B_2 for the respective spans, are:

For the Main Span:

$$B_1 = (Y_L + Y_R);\; B_2 = (Y_L - Y_R) \ldots\ldots (20b)$$

For the Left Side Span:

$$B_1 = B_2 = Y_R \ldots\ldots (20c)$$

For the Right Side Span:

$$B_1 = -B_2 = Y_R \ldots\ldots (20d)$$

These contributions, B_1 and B_2, from the individual span loads are tabulated for the various loading cases in Article 9. The contributions from all three spans must be included in ΣB_1 and ΣB_2, respectively, for substitution in Equations (19). For any span fully unloaded (stressed only by the H from temperature and from any loading in the other spans), the contributions, B_1 and B_2, are zero. For any loading in the left side span, $B_2 = B_1$; for any loading in the right side span, $B_2 = -B_1$. For any loading symmetrical about the center line of the bridge, $\Sigma B_2 = 0$ (making $T_1 = T_2$). For any span fully loaded, $G_1 = G_2\ (= p)$; hence, $Y_L = Y_R$.

For unloaded back-stays, write $n_1 = 0$ and $\frac{1}{r_1} = 0$ (or $K_1 = 0$) in the foregoing formulas. Equations (18) and (19) (the T-formulas) are written for symmetrical three-span continuous suspension bridges. For multiple-span continuous suspension bridges, and for unsymmetrical structures, new formulas for T_1 and T_2 must be written. Such formulas are derived in Article 11.

For non-continuous suspension bridges, of any number and proportions of spans, $T_1 = T_2 = 0$, and the formulas of this Article become unnecessary.

7.—EVALUATION OF H

The horizontal cable tension, H, due to any live load, p (including any supplemental dead load), and temperature change, t, all following the condition represented by the initial tension, H_w, may be evaluated as follows:

The total work, W_1, done in the vertical displacements of the suspender loads, $w + p_s$, and the cable weight, g, must equal the total work, W_2, done by the cable tension, $H_w + H$, in stretching the cable. These work quantities, W_1 and W_2, are expressed as the integrated products of the mean forces and their respective displacements, as follows, using the symbol, Σ, to denote the summation of similar expressions for all the spans:

$$W_1 = \sum \int_0^l \left(g + w + \frac{p_s}{2} \right) \eta \, dx = \sum \frac{8 f}{l^2} \left(H_w + \frac{H}{2} \right) \int_0^l \eta \, dx$$

and,

$$W_2 = \left(H_w + \frac{H}{2} \right) \left[\frac{H}{E_c A_0} L_s \pm \omega \, t \, L_t \right]$$

in which,

$$L_s = \sum \int_0^l \frac{A_0}{A_c} \frac{ds_1^3}{dx^2} \qquad \text{(21a)}$$

and,

$$L_t = \sum \int_0^l \frac{ds^2}{dx} \qquad \text{(21b)}$$

For a parabolic wire cable, having uniform A_c, it will be sufficiently accurate to write:

$$L_s = \Sigma \, l \, (\sec^3 \alpha + 8 \, n^2 \sec \alpha) \qquad \text{(21c)}$$

and,

$$L_t = \Sigma \, l \left(\sec^2 \alpha + \frac{16}{3} \, n^2 \right) \qquad \text{(21d)}$$

Similarly, for a parabolic eye-bar cable, assuming A_c varying with the slope secant, $\frac{ds}{dx}$, it will be sufficiently accurate to write Equation (21*d*) for both L_s and L_t.

Equating the foregoing expressions for W_1 and W_2, the following work equation is obtained:

$$\sum \frac{8 f}{l^2} \int_0^l \eta \, dx = \frac{H}{E_c A_0} L_s \pm \omega \, t \, L_t \qquad \text{(22)}$$

Substituting the expression for η from Equation (7), and solving for H, the basic H-equation is:

$$H = \frac{\sum K \int_0^l \left(M_0 - \frac{p_x}{c^2} \right) dx \mp r \, c^2 \, EI \, \omega \, t \, L_t + \frac{(T_1 + T_2)}{2} \Sigma' (K l)}{\sum K \left[\frac{2}{3} f \, l - \frac{l}{r c^2} - \frac{1}{H} \int_0^l (C_1 e^{cx} + C_2 e^{-cx}) \, dx \right] + r \, c^2 \frac{E}{E_c} \frac{I \, L_s}{A_0}} \qquad \text{(23)}$$

in which, E is the modulus of elasticity of the truss material and E_c that of the cable.

The summations, Σ, in Equation (23) embrace the corresponding expressions for all the spans; the symbol, Σ', likewise denotes a summation of corresponding expressions for all the spans, except that the side-span contributions are to be multiplied by $\frac{1}{2}$ before adding them.

The coefficient, K, occurring in the summations, denotes the ratio of $\frac{f}{l^2}$ for any span to $\frac{f}{l^2}$ of the main span; hence, $K = 1$ for the main span and $K_1 = \frac{r}{r_1}$ for the side spans. (Generally the ratio, K_1, for suspended side spans is between 1.00 and 1.05, representing the ratio of side-span weight to main-span weight per unit length.) For "unloaded" or "straight" back-stays, $K_1 = 0$, and all side-span terms vanish from the summations in Equation (23).

It should be noted that some approximations are involved in the foregoing derivation of the H-equation (Equation (23)), but the effect on the resulting stresses is found to be practically negligible.

In Equation (23) only the last term in the numerator represents the effect of continuity. With this term omitted (since $T_1 = T_2 = 0$ for hinges at the towers), Equation (23) reduces to the basic H-equation for non-continuous suspension bridges.

For any given loading condition, substitute the appropriate values of M_0, p_x, C_1, and C_2 in the integration terms of Equation (23), carry the two integrations over all span segments, and then re-solve the equation for H. This procedure yields the following general formula for H for all loading cases:

$$H = \frac{\Sigma\,(KA) + \frac{(T_1 + T_2)}{2} F \mp r c^2 E I \omega t L_t}{\frac{2}{3} \Sigma\,(K f l) - \sum \left(\frac{K L_c}{r c^2} \right) + r c^2 \frac{E I}{E_c A_0} L_s} \qquad (24)$$

in which, A for each span is a function of the loading in that span. A general expression for A for any loading in any span is:

$$A = \int_0^l M_0 \, dx - \frac{1}{c^2} (Y_L + Y_R) \qquad (25)$$

Values of A for different loading cases are tabulated in Article 9. After deriving the expression for A for any segmental loading (such as Case III, Article 9) by the foregoing procedure, the expressions for all other loading cases may be written by simple processes of substitution, addition, and subtraction. Use can be made of the fact that the expression for A or $\Sigma\,(KA)$ (although not its value) is algebraically additive for combinations of loading.

In Equation (24) there is introduced the abbreviation:

$$L_c = l - \frac{2}{c}\,\frac{(e^{cl}-1)}{(e^{cl}+1)} = l - \frac{2\,d}{c} = \text{(approx.)}\; l - \frac{2}{c} \quad \ldots\ldots(26a)$$

and,

$$L_{c_1} = l_1 - \frac{2}{c_1}\,\frac{(e^{(c1)(l1)}-1)}{(e^{(c1)(l1)}-1)} = l_1 - \frac{2\,d_1}{c_1} \quad \ldots\ldots\ldots\ldots(26b)$$

This abbreviation will be found convenient in condensing the formulas and applications of the Deflection Theory.

All the foregoing formulas in this Article are directly applicable to symmetrical three-span suspension bridges, continuous or non-continuous; they may also be applied to non-continuous multiple-span suspension bridges, symmetrical or unsymmetrical. Applicability also to continuous multiple-span suspension bridges, symmetrical or unsymmetrical, is obtained with only a slight change in the form of the continuity term in the numerators of Equations (23) and (24) for H. This slight modification for complete generality of the H-formula is recorded in Article 11.

Substituting for $(T_1 + T_2)$ in Equation (24), the expression given by Equation (19*a*), and then re-solving for H, the working formula for H for symmetrical three-span suspension bridges, either continuous or non-continuous, is obtained as follows:

$$H = \frac{\Sigma\,(KA) - \dfrac{F}{2\,N}\,\Sigma\,B_1 \mp r\,c^2\,E\,I\,\omega\,t\,L_t}{D = \dfrac{2}{3}\,\Sigma\,(K\,f\,l) - \displaystyle\sum\left(\frac{K\,L_c}{r\,c^2}\right) + r\,c^2\,\frac{E\,I}{E_c\,A_0}\,L_s - \frac{F^2}{2\,r\,N}} \quad \ldots(27)$$

in which, N is the denominator of Equation (19*a*).

In the working formula for H, Equation (27), the first term, $\Sigma\,(KA)$, in the numerator is the load term with continuity disregarded. The second term in the numerator, containing ΣB_1, is the load term due to continuity. The remaining term in the numerator, containing t, represents the effect of temperature change; it has the minus sign for a rise in temperature above normal, and the plus sign for a drop in temperature below normal.

In the denominator of Equation (27), the fourth term represents the contribution of the geometrical constants of the structure to the correction for continuity. In both numerator and denominator, the terms due to continuity are identified by the fact that they contain F $(= \Sigma'\,K\,L_c)$. For application to non-continuous suspension bridges, these terms vanish.

It is of interest to note that unsymmetrically loaded spans are reversible (left to right) without affecting the value of $\Sigma\,(KA)$. Furthermore, an unsymmetrically loaded main span is reversible (left to right) and unequally loaded side spans are interchangeable without affecting the value of H, since the shifting of any load to a symmetrical position about the center line of the entire structure does not alter the value of $(T_1 + T_2)$ which is represented by ΣB_1; but an unsymmetrically loaded side span cannot be reversed about its own center line without altering the value of B_1 and, therefore, of H. (This last distinction does not apply to non-continuous suspension bridges.)

The denominator of the H-formula (Equation (27)) will be denoted by the symbol, D. Although the expression for D remains unchanged for all loading conditions, it contains the variable, c, which depends upon H, and, therefore, the numerical value of D varies with the loading. The calculations for a given structure are facilitated by computing in advance the values of D for varying values of H, and tabulating or plotting the results for reference in the subsequent computations. (For an illustration, see Table 2, in Article 13.)

8.—Loading Conditions for Maximum Moments and Shears

In the continuous suspension bridge, loading conditions for maximum stress are not as uniform and definite as in the non-continuous structure; nor are they as easily predictable from a study of the basic equations. A study of the loading conditions that produced maximum calculated moments and shears for continuous suspension bridges that have been designed yields the guiding indications for loading placement that are tabulated in Fig. 2.

LOADINGS FOR MAXIMUM MOMENTS AND SHEARS

Fig. 2

The load lengths for maximum positive and negative moments and for shears generally have to be found by trial in the Deflection Theory. Generally, three trial values of the load length suffice to determine the maximum value of the function sought. In Fig. 2 the Roman numerals over the loading diagrams in the individual spans refer to the numbered designations of the respective basic loading cases tabulated in Article 9. Thus, for instance, the load condition shown in Fig. 2(a) for maximum positive moment at $0.1\,l$ is resolved into the elementary Loading Cases VII, II, and VIII, the formulas for which are listed in Article 9.

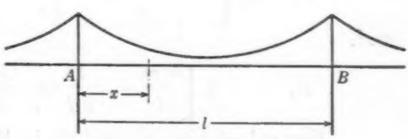
The symbols, $+t$ and $-t$, on the loading diagrams indicate whether a rise or fall in temperature should be assumed in conjunction with the indicated loading to yield the maximum value of the stress sought. A convenient rule, in practical application, is generally to use the highest temperature (in conjunction with the appropriate live-load placement) to obtain the absolute maximum M (disregarding sign) at any point of any span, and the lowest temperature (with the complementary live-load placement) to obtain the maximum of opposite sign. It may also be shown that the division points at which the temperature moment changes sign are, approximately, at $x = 0.51\,l$ and $0.85\,l$ in the main span and at $x_1 = 0.65\,l_1$ in the side spans. For sections near the towers, between the adjacent division points, use the lowest temperatures for maximum positive moment; for all other sections of the main and side spans, use the highest temperature for maximum positive moment.

In the case of structures of unusual geometric proportions, the loading diagrams in Fig. 2 may be subject to some modification. A preliminary analysis by the Elastic Theory will generally indicate the proper loading placements, as well as the trial values for the load lengths. The ten component loading cases given in Article 9 are fundamental, and are expected to cover all conditions arising in practical application.

9.—Working Formulas for Various Loadings

By considering each span separately, the various loading conditions that are useful in design (as illustrated in Article 8) may be resolved into ten primary loading cases: Six for main-span conditions, and four for side-span conditions. These ten cases, in combination, cover all loading conditions of practical importance; and they are presented in complementary pairs as follows:

Case I.—No Load in Main Span.—



Case *I*

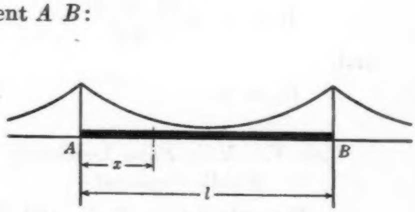
For calculating H, T_1, and T_2:

$$A = B_1 = B_2 = 0$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = 0$$

Case II.—Main Span Fully Loaded.—



Case *II*

For calculating H, T_1, and T_2:

$$A = \frac{p\,l^2}{12} - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left(l - \frac{2\,d}{c}\right) = p\,L_c$$

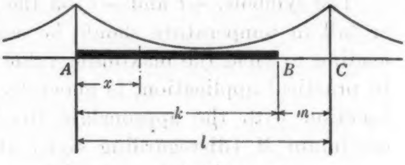
and,

$$B_2 = 0$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = p$$

Case III.—Main Span Loaded from Left End.—



Case III

For calculating H, T_1, and T_2:

$$A = p\,\frac{k^2}{12}\,(3\,l - 2\,k) - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[k - \frac{d}{c} - \frac{(1-d)}{2\,c}\,(e^{ck} - e^{cm})\right]$$

and,

$$B_2 = p\left[\frac{k\,m}{l} - \frac{1}{c\,d} + \frac{(1-d)}{2\,c\,d}\,(e^{ck} + e^{cm})\right]$$

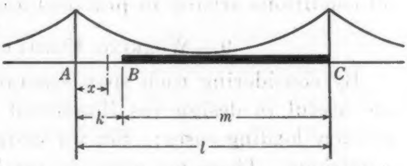
For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = \frac{p}{4\,d}\left[(1+d)\,(e^{cm} + e^{-cm}) - 2\,(1-d)\right]$$

and,

$$G_3 = p$$

Case IV.—Main Span Loaded from Right End.—



Case IV

For calculating H, T_1, and T_2:

$$A = p\,\frac{m^2}{12}\,(3\,l - 2\,m) - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[m - \frac{d}{c} + \frac{(1-d)}{2\,c}\,(e^{ck} - e^{cm})\right]$$

and,

$$B_2 = -\,p\left[\frac{k\,m}{l} - \frac{1}{c\,d} + \frac{(1-d)}{2\,c\,d}\,(e^{ck} + e^{cm})\right]$$

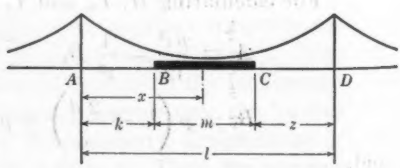
For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = -\,p\,\frac{(1+d)}{4\,d}\,(e^{cm} + e^{-cm} - 2)$$

and,

$$G_3 = 0$$

Case V.—Main Span Loaded in Middle Segment.—



Case V

For calculating H, T_1, and T_2:

$$A = p\,\frac{m}{4}\left[k\,(l-k) + z\,(l-z) + \frac{m^2}{3}\right] - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[m - \frac{(1-d)}{2\,c}\,(e^{cm} - 1)\,(e^{ck} + e^{cz})\right]$$

and,

$$B_2 = -\,p\left[m\,\frac{k-z}{l} - \frac{(1-d)}{2\,c\,d}\,(e^{cm} - 1)\,(e^{ck} - e^{cz})\right]$$

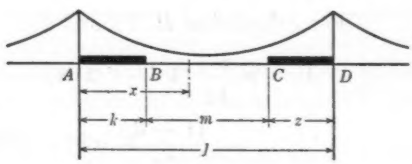
For calculating C_1 and C_2 in Segment $B\,C$:

$$G_1 = -\,\frac{p}{4\,d}\left[(1-d)\,(e^{ck} + e^{-ck}) - (1+d)\,(e^{cz} + e^{-cz})\right]$$

and,

$$G_3 = \frac{p}{2}\,(e^{ck} + e^{-ck})$$

Case VI.—Main Span Loaded in End Segments.—



Case VI

For calculating H, T_1, and T_2:

$$A = p\,\frac{k^2}{12}\,(3\,l - 2\,k)$$

$$+\,p\,\frac{z^2}{12}\,(3\,l - 2\,z) - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[(k+z) - \frac{2\,d}{c} + \frac{(1-d)}{2\,c}\,(e^{cm} - 1)\,(e^{ck} + e^{cz})\right]$$

and,

$$B_2 = p\left[m\,\frac{k-z}{l} - \frac{(1-d)}{2\,c\,d}\,(e^{cm} - 1)\,(e^{ck} - e^{cz})\right]$$

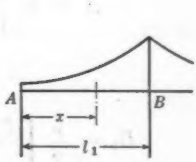
For calculating C_1 and C_2 in Segment $B\,C$:

$$G_1 = \frac{p}{4\,d}\left[(1-d)\,(e^{ck} + e^{-ck} - 2) - (1+d)\,(e^{cz} + e^{-cz} - 2)\right]$$

and,

$$G_3 = -\,\frac{p}{2}\,(e^{ck} + e^{-ck} - 2)$$

Case VII.—No Load in Side Span.—



Case VII

For calculating H, T_1, and T_2:

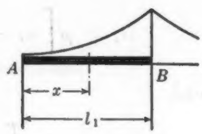
$$A = B_1 = B_2 = 0$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = 0$$

Case VIII.—Side Span Fully Loaded.—

For calculating H, T_1, and T_2:



Case *VIII*

$$A = p\,\frac{l^3_1}{12} - \frac{2}{c^2_1}\,B_1$$

$$B_1 = \frac{p}{2}\left(l_1 - \frac{2\,d_1}{c_1}\right) = \frac{p}{2}\,L_{c1}$$

and,

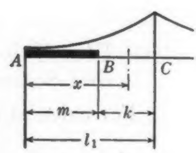
$$B_2 = \pm\,B_1$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = p$$

Case IX.—Side Span Loaded from Free End.—

For calculating H, T_1, and T_2:



Case *IX*

$$A = p\,\frac{m^2}{12}\,(3\,l_1 - 2\,m) - \frac{p}{c^2_1}\left[m - \frac{d_1}{c_1} - \frac{(1 - d_1)}{2\,c_1}\,(e^{c1m} - e^{c1k})\right]$$

$$B_1 = \frac{p}{2}\left[\frac{m^2}{l_1} - \frac{(1 - d^2_1)}{2\,c_1\,d_1}\,(e^{c1m} + e^{-c1m} - 2)\right]$$

and,

$$B_2 = \pm\,B_1$$

For calculating C_1 and C_2 in Segment $B\,C$:

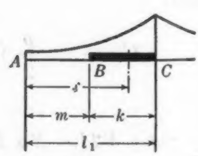
$$G_1 = \frac{p}{4\,d_1}\,(1 - d_1)\,(e^{c1m} + e^{-c1m} - 2)$$

and,

$$G_3 = -\,\frac{p}{2}\,(e^{c1m} + e^{-c1m} - 2)$$

Case X.—Side Span Loaded from Tower End.—

For calculating H, T_1, and T_2:



Case *X*

$$A = p\,\frac{k^2}{12}\,(3\,l_1 - 2\,k) - \frac{p}{c^2_1}\left[k - \frac{d_1}{c_1} - \frac{(1 - d_1)}{2\,c_1}\,(e^{c1k} - e^{c1m})\right]$$

$$B_1 = \frac{p}{2}\left[\frac{l^2_1 - m^2}{l_1} - \frac{2d_1}{c_1} + \frac{(1 - d^2_1)}{2\,c_1\,d_1}\,(e^{c1m} + e^{-c1m} - 2)\right]$$

and,

$$B_2 = \pm\,B_1$$

For calculating C_1 and C_2 in Segment $B\ C$:

$$G_1 = -\frac{p}{4\ d_1}\left[(1 - d_1)\ (e^{c1m} + e^{-c1m}) - 2\ (1 + d_1)\right]$$

and,

$$G_2 = \frac{p}{2}\ (e^{c1m} + e^{-c1m})$$

For each primary loading case, the pertinent working formulas are given in Cases *I* to *X* for the following load functions:

A = a load term in the numerator of the H-formula (Equation (24), or Equation (27)). The values of A contributed by the three spans are to be combined as $\Sigma\ (KA)$ for substitution in the H-formula. (The expressions for A are independent of continuity, and are the same for two-hinged suspension bridges.)

B_1 = a load term occurring as $\Sigma\ B_1$ in the continuity term of the numerator of the H-formula (Equation (27)), and in the numerator of the formula for $T_1 + T_2$ (Equation (19*a*)). The values of B_1 contributed by the three spans are combined as $\Sigma\ B_1$. (B_1 also occurs as a convenient abbreviation in all but the last two formulas for A.)

B_2 = a load term used in the numerator, ΣB_2, of the formula for $T_1 - T_2$ (Equation (19*b*)). The values of B_2 contributed by the three spans are to be combined as $\Sigma\ B_2$. (B_1 and B_2 are also useful in the T-formulas of Article 11 for multiple-span and unsymmetrical structures.)

G_1 = a load term used in the general working formula (Equation (12*a*) or Equation (13*a*)) for the integration constant, C_1. (Formulas for G_2, not tabulated, may be written if desired by simply changing the sign of d wherever it occurs in the formulas for G_1.)

G_2 = a load term used in the formula for $C_1 + C_2$ (Equation (12*c*), or Equation (13*c*)).

10.—Simplified Formulas for Spans Fully Loaded or Unloaded

Let p, p_1, and p_2 be the intensities of loading covering the main span, the left side span and the right side span, respectively. Then, by simply writing zero for one or more of these loading intensities, the fully unloaded condition of the corresponding span or spans is represented.

Introduce the following abbreviations for the three respective spans:

$$S = p - \frac{H}{r} \qquad \text{(28}a\text{)}$$

$$S_1 = p_1 - \frac{H}{r_1} \qquad \text{(28}b\text{)}$$

and,

$$S_2 = p_2 - \frac{H}{r_2} \qquad \text{(28}c\text{)}$$

As thus defined, S in any span represents, approximately, the uniform load actually carried by the stiffening truss.

For any condition of loading, the general formula for moments (Equation (9)) may be written in the form:

$$M = \frac{1}{c^2} S - (C_1 e^{cx} + C_2 e^{-cx}) \qquad (29)$$

provided p_x is used in the formula for S.

For the main span fully loaded or unloaded (Case I or Case II), the constants of integration become:

$$C_1 = \frac{1-d}{2}\left(\frac{S}{c^2} - T_1\right) + \frac{1-d^2}{4d}(T_1 - T_2) \qquad (30a)$$

and,

$$C_2 = -C_1 + \left(\frac{S}{c^2} - T_1\right) \qquad (30b)$$

and those for the left side or right side span fully loaded or unloaded (Case VII, or Case VIII), become:

$$C_1 = \frac{1-d_1}{2}\frac{S_{1,2}}{c^2_1} - \frac{1-d^2_1}{4d_1} T_{1,2} \qquad (30c)$$

and,

$$C_2 = -C_1 + \frac{S_{1,2}}{c^2_1} \qquad (30d)$$

Introduce the abbreviations:

$$h = 1 - \sqrt{1-d^2} \qquad (31a)$$

and,

$$h_1 = 1 - \sqrt{1-d^2_1} \qquad (31b)$$

Substitution of Equations (30) in the general formula for moments (Equation (29)), yields the following simplified formulas:

At $x = 0.5\, l$,

$$M = \frac{h}{c^2} S + (1-h)\frac{T_1 + T_2}{2} = \text{(approx.)}\ \frac{h}{c^2} S \qquad (32a)$$

At $x_1 = 0.5\, l_1$,

$$M = \frac{h_1}{c^2_1} S_{1,2} + (1-h_1)\frac{T_{1,2}}{2} = \text{(approx.)}\ \frac{h_1}{c^2_1} S_{1,2} \qquad (32b)$$

The approximate value for M in the main span is correct within 2%, on the safe side. Hence, $\frac{h}{c^2} S$ represents, very closely, the total bending moment at the middle of the main span. The approximate value for M in the side span is less permissible, although it is also on the safe side. For a non-continuous structure, both approximate values become exact.

Substituting Equations (30) in the general formula for deflections (Equation (7)), or substituting Equations (32) in Equation (6), the following simplified formulas are obtained for mid-span deflections, Δf and $\Delta f_{1,2}$:

$$E I c^2 \,\Delta f = \left(\frac{l^2}{8} - \frac{h}{c^2} \right) S + \frac{h}{2} \,(T_1 + T_2) \;\ldots\ldots\ldots\ldots (33a)$$

and,

$$E I c^2 \,\Delta f_{1,2} = \left(\frac{l^2_1}{8} - \frac{h_1}{c^2_1} \right) S_{1,2} + \frac{h_1}{2}\, T_{1,2} \;\ldots\ldots\ldots\ldots (33b)$$

These are convenient working formulas for computing maximum deflections in any span of any suspension bridge. (For non-continuous structures, the T-terms vanish and only the term containing S remains in each expression.)

Correspondingly simple formulas for shears may be written by substituting Equations (30) in Equation (10). Only the following approximate values are here presented (correct within a fraction of 1%):

At $x = 0$, or l,

$$V = \text{(approx.)} \pm \left(\frac{S}{c} - c\, T_{1,2} \right) \ldots\ldots\ldots\ldots\ldots (34a)$$

At $x_1 = l_1$,

$$V = \text{(approx.)} - \left(\frac{d_1}{c_1} S_{1,2} - c_1\, T_{1,2} \right) \ldots\ldots\ldots\ldots (34b)$$

Equations (28) to (34), inclusive, are correct in any span fully loaded or unloaded, no matter what the loading may be (full, zero, or partial) in the other spans. This fact increases the usefulness of the foregoing formulas in practical application. These equations are directly applicable to three-span and multiple-span suspension bridges, whether continuous or non- continuous, symmetrical or unsymmetrical. The formulas that follow are written only for symmetrical, three-span, continuous structures. For the loading under consideration (each span either fully loaded or unloaded), Equations (19) yield the following formulas for evaluating T_1 and T_2:

$$(T_1 + T_2) = \frac{\dfrac{H}{r} F - \Sigma' \,(p\, L_c)}{N} = -\frac{1}{N} \,\Sigma' \,(S\, L_c) \;\ldots\ldots (35a)$$

$$(T_1 - T_2) = -\frac{\frac{1}{2}\,(p_1 - p_2)\, L_c}{N + b} \;\ldots\ldots\ldots\ldots (35b)$$

or, approximately,

$$T_{1,2} = -\frac{1}{2\,N} \left[S\, L_c + S_{1,2}\, L_{c1} \right] \ldots\ldots\ldots\ldots (36)$$

(This approximate value is exact for symmetrical loading.) Substituting in Equation (27) the values of $\Sigma\,(KA)$ and $\Sigma\, B_1$ for spans fully loaded or unloaded,

$$H = \frac{\displaystyle\sum K \left(\frac{p\, l^3}{12} - \frac{1}{c^2}\, p\, L_c \right) - \frac{F}{2\,N}\, \Sigma' \,(p\, L_c) \mp r\, c^2\, E\, I\, \omega\, t\, L_t}{D} \ldots (37)$$

For the effect of temperature variation with no load on the spans (Cases I and VII), $p = 0$, $p_1 = 0$, $p_2 = 0$, $A = 0$, $B_1 = 0$, $B_2 = 0$, $G_1 = 0$, $G_2 = 0$, and $G_3 = 0$, and Equations (28) to (37) are further simplified.

11.—Application to Multiple-Span Suspension Bridges and Unsymmetrical Structures

For non-continuous three-span and multiple-span suspension bridges, whether symmetrical or unsymmetrical, Equations (1) to (37), inclusive, are completely valid and sufficient, without any modification. (For any non-continuous structure, simply omit the recognizable terms representing continuity.)

For continuous multiple-span and unsymmetrical suspension bridges, all these formulas are directly applicable except those for T_1 and T_2 and the continuity terms in the H-formulas.

All formulas in Article 4 ("Fundamental Equations") and in Article 5 ("Integration Constants") remain valid without modification for unsymmetrical or multiple spans. The formulas established for the "side spans" of the three-span bridge should be applied to the "end spans" of the multiple-span structure, and those for the "main span" should be applied to each "intermediate span" of the multiple-span suspension bridge.

In Article 7 ("Evaluation of H") all formulas remain unmodified except the continuity terms in the numerator and denominator of the H-equations. In Equation (23), instead of $\frac{(T_1 + T_2)}{2} \sum{}' (K\,l)$ write $\sum \left(\frac{T_L + T_R}{2} K\,l \right)$. In Equation (24), instead of $\frac{(T_1 + T_2)}{2} F = \frac{(T_1 + T_2)}{2} \sum{}' (K\,L_c)$ write $\sum \left(\frac{T_L + T_R}{2} K\,L_c \right)$. With these slight modifications, the continuity terms are generalized to cover multiple-span as well as unsymmetrical three-span structures. For evaluating H in multiple-span and unsymmetrical suspension bridges, Equation (24), as thus modified for complete generality, should be used instead of Equation (27) and its derivative, Equation (37), since the continuity terms in the latter formulas are valid only for symmetrical three-span structures and would be replaced by more complicated expressions for the general case.

All the working formulas tabulated in Article 9 ("Loading Cases") remain valid without modification for unsymmetrical or multiple spans as well as all the formulas through Equations (34) in Article 10 ("Simplified Formulas").

The formulas for T_1 and T_2 presented in Article 6 (Equations (18) and (19)), and the derived simplified T-formulas in Article 10 (Equations (35) and (36)) are valid only for symmetrical three-span structures. For the more general case of multiple-span suspension bridges, of any number and proportions of spans, new T-formulas are needed and are presented hereinafter. (Note that Equations (20) are general and remain valid.)

Following the procedure of Article 6, let the break in slope at the first, second, third, etc., towers (before neutralization by the tower moments, T, and

omitting the uniform factor, $EI\ c^2$, be denoted by U_1, U_2, U_3, etc. Then (compare Equation (18*c*)) the expression for U at any tower, say, at the second tower, is of the general form:

$$U_2 = (R_{2R} + R_{3L}) - c_2\,(C_1 e^{(c2)(l2)} - C_2 e^{-(c2)(l2)})_{2R} + c_3\,(C_1 - C_2)_{3L} - 4\,H\,(n_2 + n_3) \quad \text{.........(38)}$$

In Equation (38) the numeral and letter subscripts have the significance explained in Article 6. Advance the numeral subscripts in writing the similar expressions for U at the successive towers. (The expression for U at the last tower would be similarly written, if x in the last span were measured from left to right; otherwise, the third term has the reversal modification shown in Equation (18*d*)).

The generalized basic formulas for T_1, T_2, etc., at the n successive towers are then as follows:

$$\left.\begin{array}{l} \dfrac{T_1 - 0}{l_1} - \dfrac{T_2 - T_1}{l_2} = U_1 \\[2ex] \dfrac{T_2 - T_1}{l_2} - \dfrac{T_3 - T_2}{l_3} = U_2 \\[2ex] \cdots\cdots\cdots\cdots\cdots\cdots \\[2ex] \dfrac{T_n - T_{n-1}}{l_n} - \dfrac{0 - T_n}{l_{n+1}} = U_n \end{array}\right\} \quad \text{.........(39)}$$

Since there are as many of these equations as there are towers, Equations (39) can be solved for the n-values of T. (It is interesting to note that the functions, U and T, appear in Equations (39) in exactly the same relationship as concentrated loads and bending moments, respectively, on a simple beam of span, $\Sigma\, l$. Consequently, a solution of Equations (39) for the value of T at any tower may be written directly as the bending moment at that point producible by the functions, U, applied as concentrated loads at the respective tower points on the span, $\Sigma\, l$.)

In Equations (39), substitute the respective values of U as defined by Equation (38), utilizing the relations given by Equations (12), (13), and (20*a*). In this way the following generalized working formulas for T, for any number of spans of any proportions, are obtained:

$$\left.\begin{array}{l} 0 - (a_1 + a_2)\quad T_1 + b_2\,T_2 = 2\,(Y'_{1R} + Y'_{2L}) \\ b_2\,T_1 - (a_2 + a_3)\quad T_2 + b_3\,T_3 = 2\,(Y'_{2R} + Y'_{3L}) \\ b_3\,T_2 - (a_3 + a_4)\quad T_3 + b_4\,T_4 = 2\,(Y'_{3R} + Y'_{4L}) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ b_n\,T_{n-1} - (a_n + a_{n+1})\,T_n + 0 = 2\,(Y'_{nR} + Y'_{(n+1)R}) \end{array}\right\} \quad \text{.....(40)}$$

Equations (40) are applicable, with complete generality, to all types of continuous suspension bridges considered in this paper, whether symmetrical or unsymmetrical, three-span or multiple-span, with tie cables or without tie cables, and with or without tower resistance to be considered.

In writing Equations (40), the following abbreviations are used: In any span,

$$a = cd + \frac{c}{d} - \frac{2}{l} \quad \text{.....................(41}a\text{)}$$

and,

$$b = -cd + \frac{c}{d} - \frac{2}{l} \quad \text{.....................(41}b\text{)}$$

At either end of any span,

$$Y'_{L,R} = Y_{L,R} - \frac{H L_c}{2r} \quad \text{.....................(42)}$$

(Note that the subscripts, L and R, refer to the left and right ends, respectively, of any span; except that, in the last span, where the direction of measuring x is reversed, Y'_R and Y_R refer to the left or tower end.) By Equations (20), in any intermediate spans,

$$Y'_{L,R} = \frac{1}{2}(B_1 \pm B_2) - \frac{H L_c}{2r} \quad \text{.................(42}a\text{)}$$

and in either end span,

$$Y'_R = B_1 - \frac{H L_c}{2r} \quad \text{.....................(42}b\text{)}$$

Hence, in any span fully loaded,

$$Y'_L = Y'_R = \frac{1}{2} L_c S \quad \text{.................(42}c\text{)}$$

and in any unloaded span,

$$Y'_L = Y'_R = -\frac{H L_c}{2r} \quad \text{.................(42}d\text{)}$$

For a symmetrical three-span continuous suspension bridge, Equations (39) reduce to Equations (18), and Equations (40) reduce to Equations (19). For an unsymmetrical three-span continuous suspension bridge (as well as for any three-span continuous suspension bridge with tie cables), Equations (40) reduce to:

$$\left.\begin{aligned} T_1 &= -2\,\frac{(a + a_3)(Y'_{1R} + Y'_{2L}) + b\,(Y'_{2R} + Y'_{3R})}{(a + a_1)(a + a_3) - b^2} \\ \text{and,}\quad & \\ T_2 &= -2\,\frac{b\,(Y'_{1R} + Y'_{2L}) + (a + a_1)(Y'_{2R} + Y'_{3R})}{(a + a_1)(a + a_3) - b^2} \end{aligned}\right\} \quad \text{.....(43)}$$

Simply write $a_1 = a_3$ to obtain Equations (19) for the symmetrical case. For a symmetrical continuous four-span suspension bridge, Equations (40) reduce to the following working formulas for T:

$$\left.\begin{aligned} T_2 &= -\frac{b\,(Y'_{1R} + Y_2'_L + Y'_{3R} + Y'_{4R}) + (a + a_1)\,(Y'_{2R} + Y'_{3L})}{a\,(a + a_1) - b^2} \\ \frac{1}{2}\,(T_1 + T_3) &= -\frac{a\,(Y'_{1R} + Y'_{2L} + Y'_{3R} + Y'_{4R}) + b\,(Y'_{2R} + Y'_{3L})}{a\,(a + a_1) - b^2} \\ \frac{1}{2}\,(T_1 - T_3) &= -\frac{(Y'_{1R} - Y'_{4R}) + (Y'_{2L} - Y'_{3R})}{(a + a_1)} \end{aligned}\right\} \ldots.(44)$$

With the aid of Equations (42*a*) and (42*b*), these working formulas may be re-written in terms of B_1 and B_2 which are tabulated in Article 9. If the successive spans are fully loaded with uniform loads of p_1, p_2, p_3, and p_4, respectively (where any or all of these loads may be zero, as in Article 10), Equations (43) and (44) may be re-written in simple form in terms of S_1, S_2, S_3, and S_4, upon substituting Equation (42*c*).

Always starting from the completely general Equations (40) (in the same manner as Equations (43) and (44) were written for the three-span and four-span cases, respectively) similar working formulas may be written for other cases of continuous multiple-span or unsymmetrical suspension bridges, of any number of spans.

The formulas in the preceding Articles included non-continuous suspension bridges in general, and symmetrical three-span continuous structures. With the T-formulas (and modifications of the H-formulas) presented in this Article, the Deflection Theory is further generalized and extended to cover also multiple-span and unsymmetrical continuous suspension bridges.

12.—Application to Multiple-Span Suspension Bridges with Tie Cables

Multiple-span suspension bridges may be increased in efficiency by the provision of tie cables connecting the tower tops. The function of the tie cables is to reduce relative tower-top movements or span elongations, $\Delta\, l$. The idea was invented about 75 years ago, and, more recently, several structures of this type have been built in France and projected in the United States; but an accurate analysis for this type of bridge has been lacking thus far. This practical problem affords an opportunity to show the comprehensiveness and convenience of the "Generalized Deflection Theory" as a foundation for further extensions of suspension-bridge theory. Solution of the problem of analysis is, of course, a prerequisite to correct economic evaluation and application of a structural type.

For multiple-span suspension bridges with tie cables, all the formulas in the preceding Articles remain valid except the formulas for H and some of the formulas for T (namely, the T-formulas written for symmetrical structures). These formulas require some modification when tie cables are used, because H then varies from span to span. All other formulas of this paper remain directly applicable without change, including specifically all the formulas in Article 4 ("Fundamental Equations"), Article 5 ("Integration Constants"), and Article 9 ("Working Formulas"), as well as Equations (28) to (34), inclusive, of Article 10 ("Simplified Formulas"), and Equations (39) to (43), inclusive, of Article 11 ("Multiple Spans and Unsymmetrical Structures").

The modified procedure for determining H in each span is, as follows: Let H_0 be the initial horizontal tension in the tie cable, corresponding to H_w in the main cable. Then $(H_0 + H_w)$ is the same in all spans (except for a minor correction, usually negligible, for any initial stress in the towers in the direction of cable tension). Let H_T be the additional horizontal tension induced in the tie cable (by span elongation and temperature change), corresponding to H (due to live load, span elongation, and temperature change) in the main cable. Then $(H + H_T)$ has the same value in all spans, except for a minor correction due to the deflection resistance of the towers. If j is the resistance to unit deflection of the tower top,

$$(H + H_T)_1 = (H + H_T)_2 - j_1 \Delta l_1$$
$$= (H + H_T)_3 - j_1 \Delta l_1 - j_2 (\Delta l_1 + \Delta l_2) = \ldots \text{ etc.} \qquad (45)$$

The subscripts designate the numbers of the successive spans and of the successive towers, counting from one end of the bridge.

From the geometry of the tie cable before and after deflection, one may write with sufficient accuracy:

$$H_T = P \Delta l + X \qquad (46)$$

in which,

$$P = \frac{E_T A_T}{(L_s)_T} \qquad (46a)$$

and,

$$X = E_T A_T \left[\frac{8}{3} n^2_T (R^2_0 - 1) - \omega t \right] \qquad (46b)$$

In Equations (46*a*) and (46*b*), the subscripts, T, designate tie-cable quantities, and,

$$R_0 = \frac{H_0}{H_0 + H_T} \qquad (46c)$$

To write the corresponding H-formula for the main cable, follow the procedure of Article 7, equating external and internal work, but limit consideration to a single span. In this case, the expression for external work, W_1, contains the additional term, $\left(H_w + \frac{H}{2} \right) \Delta l$, representing the work done against cable tension by the span elongation, Δl. With this modification, Equation (24) takes the form:

$$H = Q \Delta l + Z \qquad (47)$$

in which,

$$Q = \frac{1}{D'} (r c^2 E I) \qquad (47a)$$

and,

$$Z = \frac{1}{D'}\left[A + \frac{L_c}{2}(T_L + T_R) - r\,c^2\,E\,I\,\omega\,t\,L_t\right] \qquad \text{(47b)}$$

In Equations (47*a*) and (47*b*), D' is the denominator of the H-formula for a single span:

$$D' = \frac{2}{3}\,f\,l - \frac{L_c}{r\,c^2} + r\,c^2\,\frac{E\,I}{E_c\,A_0}\,L_s \qquad \text{(47c)}$$

In Equations (46*a*) and (47*b*), L_s and L_t are the values for a single span. From Equations (46) and (47) combined, the elongation of any span is given by,

$$\Delta\,l = \frac{(H + H_T) - (X + Z)}{(P + Q)} \qquad \text{(48)}$$

The values of Δl for all the spans must always satisfy the primary governing conditions:

$$\Sigma\,(\Delta l) = 0 \qquad \text{(48a)}$$

Upon substituting Equations (45) and (48), Equation (48*a*) yields:

$$(H + H_T)_1 = \frac{\sum\left(\dfrac{X + Z}{P + Q}\right) - \sum\left(\dfrac{J\,\Delta l}{P + Q}\right)}{\sum\left(\dfrac{1}{P + Q}\right)} \qquad \text{(49)}$$

which is the key formula for the solution of multiple-span suspension bridges with tie cables. In Equation (49),

$$\sum\left(\frac{J\,\Delta l}{P + Q}\right) = \Delta l_1\left[\frac{j_1}{(P + Q)_2} + \frac{(j_1 + j_2)}{(P + Q)_3} + \frac{(j_1 + j_2 + j_3)}{(P + Q)_4} + \ldots\right]$$

$$+ \;\Delta l_2\left[\frac{j_2}{(P + Q)_3} + \frac{(j_2 + j_3)}{(P + Q)_4} + \ldots\right] + \Delta l_3\left[\frac{j_3}{(P + Q)_4} + \ldots\right] + \ldots \qquad \text{(49a)}$$

Equation (49*a*) is a minor term in Equation (49) and is usually negligible, The other quantities in Equation (49) are the simple summations of the respective indicated expressions for all the spans.

If the elastic resistances or deflections of the towers are negligible, or if rocker towers are used, one may write $j = 0$ and Equations (45) and (49) reduce to:

$$(H + H_T)_1 = (H + H_T)_2 = (H + H_T)_3 = \ldots = \frac{\sum\left(\dfrac{X + Z}{P + Q}\right)}{\sum\left(\dfrac{1}{P + Q}\right)} \qquad \text{(50)}$$

In any case, Equation (50) may be used as a first approximation. It will generally be sufficiently accurate except in cases of rigid towers or extreme distortion of the structure.

After $(H + H_T)$ is determined for all spans by Equation (50), or for the individual spans by Equations (49) and (45), the value of Δl for any span is given by Equation (48), and the value of H for the span is then given by Equation (47). The values of M, V, η, etc., are then given by the formulas of Articles 4, 5, and 9.

The solution of the problem to this point suffices if the stiffening truss is non-continuous at the towers. In such case, simply write $T_L = T_R = 0$ in Equation (47*b*).

If the multiple-span suspension bridge with tie cables has the stiffening truss continuous at the towers, the correct expressions for T are given for any number of spans of any proportions by the general T-formulas, Equations (39) and (40) of Article 11. Equations (18) and (43) also remain valid. In the expressions for U, typified by Equation (38), a slight modification is required to provide for the varying H; instead of $4\ H\ (n_2 + n_3)$, write $4\ (H_2\ n_2 + H_3\ n_3)$. The forms in which the general T-formulas (Equations (40)) are written, as well as any T-formulas for unsymmetrical structures, automatically take care of this correction.

The formulas of this Article also cover suspension bridges without tie cables, but with deflection resistances of towers to be considered. In such cases, simply write zero for H_0, H_T, P, and X, in the foregoing formulas. If, in addition, $j = 0$, the formulas of this Article reduce to those of Article 7.

13.—Practical Application to Continuous Spans

In order to test the practical applicability of the Generalized Deflection Theory and, at the same time, to establish data for comparisons of types of structure and theories of analysis, the theory and formulas developed in this paper have been applied to the analysis of a three-span continuous suspension bridge.

Design I

The structure selected for the numerical application of the theory has an 800-ft main span and two 400-ft side spans, and had been previously designed as a two-hinged suspension bridge. The trusses have a constant depth of 12 ft throughout and are spaced 45 ft, center to center. For the first comparative design (herein referred to as Design I), the same moments of inertia, ($I = 1\,960$ in.^{2} ft^2, and $I_1 = 2\,420$ in.^{2} ft^2), were assumed as in the two-hinged design, in order to ascertain comparative rigidity under conditions of equal economy.

Comparison with Elastic Theory.—The stresses in the continuous spans were first computed by the Elastic Theory. This preliminary analysis incidentally yielded the approximate loading conditions to be used as a guide for assuming trial load lengths in the more exact analysis. The stresses were then computed more accurately by the Generalized Deflection Theory, using the formulas and procedure developed in this paper. The maximum bending moments yielded for Design I by the two respective theories are plotted in

Fig. 3. The percentage reductions obtained by the application of the Deflection Theory are also plotted. These reductions in maximum bending moments range from 11% at the tower to 50% in each span. Except for a comparatively short length close to the tower (including about one-eighth of the side span and one-twentieth of the main span) where the average reduction effected by the Deflection Theory is only about 13%, the reductions are approximately 44% in the main span and 48% in the side span. A comparison of the total areas under the bending-moment graphs for the two respective theories, as

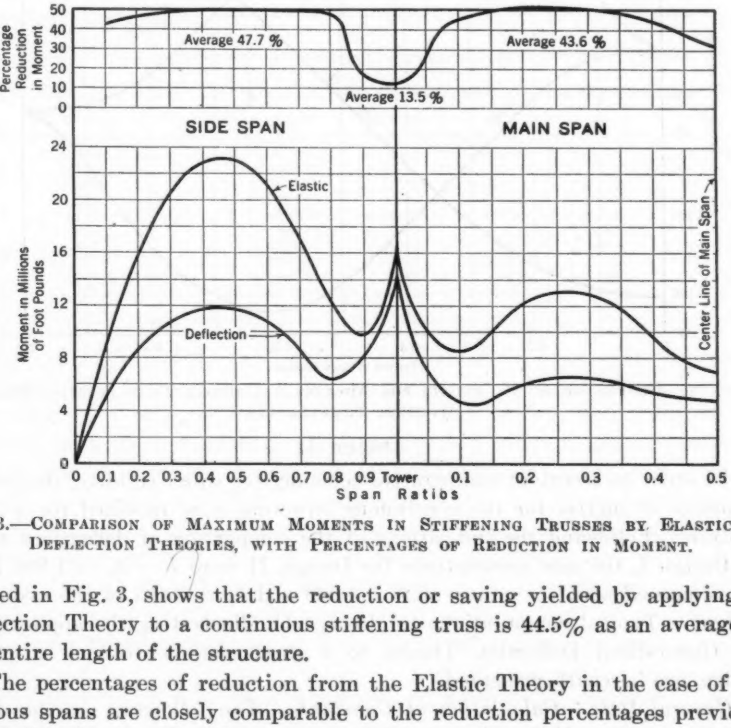


FIG. 3.—COMPARISON OF MAXIMUM MOMENTS IN STIFFENING TRUSSES BY ELASTIC AND DEFLECTION THEORIES, WITH PERCENTAGES OF REDUCTION IN MOMENT.

plotted in Fig. 3, shows that the reduction or saving yielded by applying the Deflection Theory to a continuous stiffening truss is 44.5% as an average for the entire length of the structure.

The percentages of reduction from the Elastic Theory in the case of continuous spans are closely comparable to the reduction percentages previously established for two-hinged suspension bridges. The direct application of any approximate factor of reduction, however, is modified in the continuous structure by the variation of reduction ratio near the towers.

Comparison of Deflections.—The maximum deflections in the two-hinged design, computed by the Deflection Theory (using Equations (33), with $T_1 = T_2 = 0$) were 5.16 ft in the main span and 3.30 ft in the side spans. In the continuous structure of Design I (having the same assumed moments of inertia to represent equal economy), the maximum deflections were also computed by Equations (33) and were found to be 5.28 ft in the main span and only 2.81 ft in the side spans. The comparison of maximum deflections shows a reduction of 14.9% in the side spans of the continuous design and an increase of 2.3% in the center span, or an average reduction of 6.3% for the

entire structure. Hence, for designs of equal economy, the continuous structure of 800-ft main span is, on the whole, about 6% more rigid than the two-hinged type.

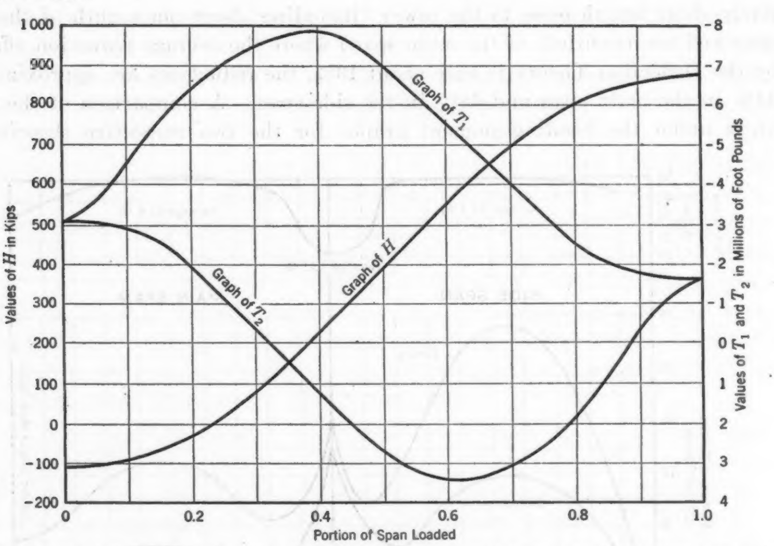


FIG. 4.—GRAPHS OF H, T_1, AND T_2 FOR ADVANCING UNIFORM LOAD IN MAIN SPAN AT HIGHEST TEMPERATURE

Design II

In order to ascertain comparative economy for equal rigidity, the assumed moments of inertia for the continuous structure were modified for a second analysis. Following the indications of the comparison of deflections yielded by Design I, the new assumptions for Design II were $I = I_1 = 1\,960$ in.^{2} ft^2 (as compared with the values of $I = 1\,960$ and $I_1 = 2\,420$ in the two-hinged design). Typical computations for Design II, illustrating the application of the Generalized Deflection Theory to a three-span continuous suspension bridge, are herewith presented.

General Data.—Calculation of Constants.—**The following are the dimensional constants: Main span: l = 64 panels = 800 ft; f = 84 ft; and n = 0.105; side spans: l_1 = 32 panels = 400 ft; f_1 = 21 ft; and n_1 = 0.0525; sec α = 1.03484; $r = r_1$ = 952.381; K_1 = 1.00; and A = 87.8 in.^{2}**

The following are the loading constants (all values per cable): Dead load: w = 3 850 lb per ft; live load: p = 1 300 lb per ft; $H_w = \dfrac{w\,l^2}{8\,f}$ = 3 667 kips; temperature: $t = \pm\ 60°$ F; E = 29 000 000; ω = 0.0000065; and, $E\,\omega\,t$ = 11 310 lb per sq in.

By Equations (21): L_s = 2 075, and L_t = 1 998. E_c = 25 000 000. The truss constants are as follows: Main span: I = 1 960 in.^{2} ft^2, and $E\,I$ = 56 840 000 ft^2 kips; side span: I_1 = 1 960 in.^{2} ft^2, and $E\,I_1$ = 56 840 000 ft^2 kips.

Calculation of Values of D.—Preparatory to obtaining values of H for various conditions of loading, the values of D (the denominator of the formula for H) were calculated by Equation (27) for different values of H from $-$ 100 to $+$ 1 200 kips. The values of d and d_1 for the various values of $c\,l$ and $c_1\,l_1$ were written directly from Table 1 (Article 5). Computations for the values of D are made by a systematic tabular method which embraces the step-by-step numerical operations. The principal constants and the values of D for the different values of H are given in Table 2. The values of D used in computing H for various loadings may be obtained for any trial H by interpolation in this tabulation.

TABLE 2.—COMPUTATION OF D.

H, in kips	$c = c_1$	$cl = 2\,c_1 l_1$	d	d_1	D
—100	0.007921	6.3368	0.99647	0.91927	24 226
0	0.008032	6.4256	0.99677	0.92262	24 600
+100	0.008141	6.5128	0.99704	0.92579	24 955
200	0.008248	6.5984	0.99728	0.92880	25 325
300	0.008354	6.6832	0.99750	0.93165	25 671
400	0.008458	6.7664	0.99770	0.93436	26 020
600	0.008664	6.9312	0.99805	0.93940	26 685
800	0.008865	7.0920	0.99834	0.94393	27 342
1 000	0.009061	7.2488	0.99858	0.94806	27 972
1 200	0.009253	7.4024	0.99878	0.95181	28 584

Values of H for Advancing Uniform Load in Main Span.—The values of H, T_1, and T_2 for varying lengths of a continuous advancing uniform live load in the main span, at highest temperature and with no load in the side spans, are necessary in the computation of the maximum positive moments at several points in the main span. By the use of Equations (27) and (19) with the formulas of Cases III and VII, the values of H, T_1, and T_2 are calculated for the aforementioned loading for successive lengths varying by 0.1 of the span. Agreement with the trial value of H is usually obtained on the second trial, and the values of T_1 and T_2 are then computed. The results are plotted in Fig. 4.

Maximum Positive Moments in Main Span.—Maximum moments are positive for Points $\frac{x}{l} = 0.2$ to 0.5, and negative for Points $\frac{x}{l} = 0$ and 0.1. Maximum positive moments for Points $\frac{x}{l} = 0.2$, 0.3, and 0.4 are calculated by Equations (9) for the load condition of Case III (as indicated in Fig. 2(*a*)). For each section, x, the value of M is calculated for three or four different trial load lengths, k, until the maximum value of M for the section is determined. The values of H, T_1, and T_2 for each load length are taken from the H and T-graphs, Fig. 4. The governing quantities, in abbreviated form, are recorded in Table 3.

Maximum Positive Moment at Mid-Span.—Maximum positive moment at the center of the main span, as indicated in Fig. 2(*a*), is produced when the central part of the span is symmetrically loaded with a uniform load for a length, m, at highest temperature and with no load on the side spans. In

order to find the value of m for which the moment at this point is a maximum, it is necessary to assume different trial values of m and calculate the corresponding values of M by use of the formulas of Case V. Before thus com-

TABLE 3.

Section: $\frac{x}{l}$	0.2	0.3	0.4
Load: k (assumed)	$0.41l$	$0.475l$	$0.53l$
H, (from Fig. 4)	242	348	440
c (by Equation (5))	0.00829	0.00841	0.00850
T_1 (from Fig. 4)	−7 840	−7 330	−6 515
T_2 (from Fig. 4)	+1 430	+2 415	+3 115
C_1 (by Equation (12a))	604	365.5	234.6
C_2 (by Equation (12b))	22 435	20 160	17 890
e^{cx}	3.764	7.53	15.18
Maximum moment, (by Equation (9))	+6 970	+7 770	+6 870

puting each M, however, it is also necessary to obtain, by trial, the value of H corresponding to the assumed value of m. The final quantities, in condensed form, are: $\frac{m}{l}$ (assumed) $= 0.33$; H (assumed trial value) $= 420$ kips; c (by Equations (5)) $= 0.00848$; $e^{cm} = 9.37$; $e^{ck} = 9.69$; D (from Table 2) $= 26\,085$; H (by Equation (27)) $= 419$ kips; T_1 (by Equations (19) and formulas of Case V) $= +\,450$ kips; C_1 (by Equation (12a) and formulas of Case V) $= 92.6$; C_2 (by Equation (12b) and formulas of Case V) $= 81\,560$; $e^{cx} = e^{0.5(cl)} = 29.71$; and, the maximum moment, M (by Equation (9)) $= +\,6\,440$ ft-kips.

Maximum Negative Moment in Main Span Near Tower.—The maximum negative moment at the Point $\frac{x}{l} = 0.1$ occurs when the adjacent side span is fully loaded at highest temperature and the other two spans are unloaded. For this condition of loading, H is determined by trial, using the formulas of Case VIII. The value of M is then calculated by the formulas of Cases I and VIII, and Equation (9), or Equation (29). A summary of the principal resulting values is as follows: H (final value by Equation (27), or Equation (37)) $= -\,87.4$ kips; c (by Equations (5)) $= 0.00793$; d (from Table 1) $= 0.99649$; T_1 (by Equations (19) and formulas of Case VIII) $= -\,11\,550$ ft-kips; T_2 (by Equations (19) and formulas of Case VIII) $= -\,1\,550$ ft-kips; C_1 (by Equation (30a)) $= 4.97$; C_2 (by Equation (30b)) $= 13\,000$; $e^{cx} = 1.886$; and, M (by Equation (29)) $= 5\,450$ ft-kips.

Maximum Negative Moment at Tower.—To obtain the maximum negative moment at the tower, it is necessary to load the adjacent side span fully and a part of the main span next to the tower, at highest temperature. The main span load length for which the tower moment is a maximum must be determined, by the formulas of Cases III and VIII, as previously outlined for the computation of maximum moment at mid-span. This computation is briefly summarized as follows: H (trial value) $= 150$ kips; c (by Equations (5)) $= 0.00819$; d (from Table 1) $= 0.99715$; d_1 (from Table 1) $= 0.92725$; $\frac{k}{l} = 0.34$;

$e^{ck} = 9.29$; $e^{cm} = 75.69$; D (from Table 2) $= 25\,140$; H (by Equation (27) and formulas of Cases III and VIII) $= 158.2$ kips; and T_1 (by Equations (19) and formulas of Cases III and VIII) $= 16\,300$ ft-kips.

Maximum Positive Moments Near Free End of Side Span.—Maximum moments are positive for all sections in the side span, except Points $\frac{x_1}{l_1} = 0.8$ and 0.9 near the tower. The loading condition producing maximum positive moments at Sections $\frac{x_1}{l_1} = 0.1$ to 0.4, is shown in Fig. 2(a) to be the same as that for negative moment at Point $\frac{x}{l} = 0.1$ in the main span, the side span being fully loaded at highest temperature, with no load on the other two spans. The computations for H, T_1, and T_2 have already been indicated for this loading condition; the remaining calculations for the determination of the moment at Point $\frac{x_1}{l_1} = 0.4$ are: C_1 (by Equation (30c)) $= 1\,374$; C_2 (by Equation (30d)) $= 20\,745$; $e^{cz} = 3.56$; and, maximum moment (by Equation (29)) $= +\,11\,410$ ft-kips.

Maximum Positive Moments Near Center of Side Span.—The maximum moment at Point $\frac{x_1}{l_1} = 0.7$ is positive and occurs when both side spans are fully loaded at lowest temperature. The maximum positive moment at Points $\frac{x_1}{l_1} = 0.5$ and 0.6 is produced by fully loading both side spans at highest temperature. The computations for the moment at mid-span are: H (trial value) $= -\,65$ kips; $c_1 = c$ (by Equations (5)) $= 0.00796$; $D = 24\,357$; H (by Equation (37)) $= -\,64.9$ kips; $T_1 = T_2$ (by Equation (36)) $= -\,9\,980$ ft-kips; C_1 (by Equation (30c)) $= 1\,273.6$; C_2 by Equation (30d)) $= 20\,315$; $e^{0.5(c_1 l_1)} = 4.91$; and M (by Equation (32b)) $= +\,11\,205$ ft-kips.

Maximum Negative Moment in Side Span Near Tower.—The loading condition for maximum negative moment at Point $\frac{x_1}{l_1} = 0.8$ is shown in Fig. 2(b) to be the same as for positive moments in the main span. This moment is computed in the same manner as for maximum positive moments at Sections $\frac{x}{l} = 0.2$, 0.3, and 0.4 in the main span, except for the use in this computation of the integration constants of Case VII.

The maximum negative moment at Point $\frac{x_1}{l_1} = 0.9$ occurs under a partial loading of the side span and the main span, at highest temperature, as shown in Fig. 2. In obtaining the maximum value of this moment, it must be computed by use of the formulas of Cases III and IX for several trial load lengths in both spans.

Minimum Moments.—The loading conditions under which minimum moments are produced in the trusses are also shown in Fig. 2. The values of these moments are obtained in the same general manner, as outlined for the calculations of maximum moments, and usually occur at lowest temperature in combination with the indicated live loadings.

Maximum Shears.—The maximum positive and negative shears are calculated for the loading conditions indicated in Fig. 2(c) and Fig. 2(d). The method of procedure is the same as that for the calculation of maximum moments, involving the assumption of trial values of H and trial load lengths to secure the values for which the shears are a maximum or minimum. The shear is computed from the values of c, C_1, and C_2 by Equations (10) and (34b).

Maximum Deflection in Main Span.—Maximum deflection in the main span is produced by loading that span fully, at highest temperature. The value of H is obtained by trial from Equation (37), with the formulas of Case II, the result having been plotted in the H-curve for advancing load in the main span (Fig. 4). The calculation of the deflection at the center of the span in condensed form is, as follows (the calculation of H, T_1, and T_2 was indicated previously): H (by Equation (37)) $= 873$ kips; $T_1 = T_2$ (by Equation (36)) $= -1\,575$ ft-kips; S (by Equation (28a)) $= 0.3845$ kips per ft; h (by Equation (31a)) $= 0.94415$; and Δf (by Equation (33a)) $= 5.44$ ft.

Maximum Deflection in Side Span.—Maximum downward side-span deflection occurs when the two side spans are both fully loaded at highest temperature. The computation of the necessary constants for this loading condition has already been indicated in the calculation of positive moment at the center of the side span. The deflection at the center of the side span may be computed from Equation (7), or from Equation (33b), and is found to be 3.10 ft.

It will be noted that the side-span deflection is 6% less than in the two-hinged design, while the main-span deflection is 5.4% greater. The average rigidity over the entire length differs from that of the two-hinged design by only 0.3 per cent.

Comparison of Maximum Moments for Continuous and Two-Hinged Designs.—The maximum bending moments produced by live load and temperature in the continuous spans of Design II, computed as herein outlined, are plotted in Fig. 5. For comparison, Fig. 5 also includes a curve of the maximum bending moments produced by live load and temperature in the corresponding two-hinged design. The saving in chord material by the adoption of the continuous type, as indicated by the percentage of difference between the moment areas under these two respective graphs, is 5%, and this value is substantiated by the actual design of the truss members.

Advantages of the Continuous Type.—The foregoing comparative design studies indicate that, for a suspension bridge of 800-ft main span, the continuous type is approximately 6% more rigid than a two-hinged design of the same economy, and about 5% more economical than a two-hinged design of the same rigidity. These percentage differences in favor of the continuous type will be greater in shorter spans or with deeper trusses.

As the length of span or its flexibility is increased, the effect of continuity at the towers is lost on the spans at points proportionately nearer and nearer the towers. It is for this reason that the advantage of the continuous type is greater with shorter spans and deeper stiffening trusses. In general, the advantage of the continuous type over the two-hinged design will be governed by the stiffness factor, $S' = \frac{1}{l}\sqrt{\frac{EI}{H_w}} = \frac{1}{l^2}\sqrt{\frac{8fEI}{w}}$, which also governs the percentage correction between the results of the Elastic and the Deflection Theories.

The continuous type has an advantage in respect to behavior under lateral forces. The lateral rigidity is greater than in the two-hinged design, and

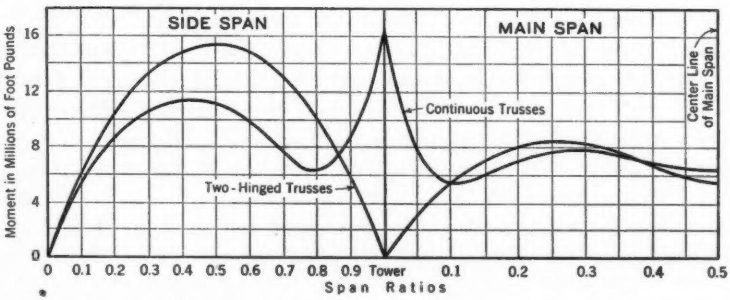


FIG. 5.—COMPARISON OF MAXIMUM MOMENTS FOR LIVE LOAD AND TEMPERATURE IN CONTINUOUS AND TWO-HINGED STIFFENING TRUSSES.

there is a better distribution and absorption of stresses from lateral loading. In fact, for spans of 800 ft, or less, for which the ratio of width to span is greater than 1:20, the chord sections in a continuous design are not affected by wind stresses, since the better distribution of these stresses brings them under the 25% increase in allowable stress permitted under this loading by the usual design specifications.

As a general conclusion, it may be stated that the continuous type of suspension bridge offers advantages over the two-hinged type for spans under 1 000 ft, designed for highway loading, and for longer spans when designed for railroad loading.

ACKNOWLEDGMENT

This paper, in different form, was originally presented by the writer at the Joint Session of the Structural Division of the Society and the Applied Mechanics Division of the American Society of Mechanical Engineers, at the Annual Convention, at Chicago, Ill., June 30, 1933. In the original paper, the continuity moments at the towers were evaluated by applying the "theorem of three moments" to the usual assumption of uniform suspender loading in each span. In his discussion, Dr. S. Timoshenko suggested the evaluation of these continuity moments by equating slopes at the towers. This sug-

gestion has proved valuable, yielding a simpler and more accurate solution and prompting a revision of the paper to the present condensed and improved form. The writer is gratefully indebted to Dr. Timoshenko for this helpful suggestion. Acknowledgment is also due C. H. Gronquist, Assoc. M. Am. Soc. C. E., for valuable assistance in the preparation of the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

IRRIGATION HYDRAULICS[1]

FINAL REPORT OF SPECIAL COMMITTEE

To the Board of Direction,
American Society of Civil Engineers:

Gentlemen.—The Committee was appointed June 30, 1922. It was discontinued December 31, 1933, at the request of the Committee itself. The reasons for this request were:

(1) The Committee has been in existence for $11\frac{1}{2}$ years and has contributed much to the advancement of the Engineering Profession during that period.

(2) The Committee has been very active throughout its existence while funds for its work were available. It has generally held two meetings per year and each member has had definite assignments of work.

(3) When funds were curtailed the work suffered to such an extent that it was thought the better policy would be for the Committee to suspend work altogether, resting on its record of accomplishments, rather than to continue a desultory existence. The Society's funds would thus be conserved rather than spent aimlessly.

The Board of Direction at its meeting of June 25, 1933, acquiesced in this view and discharged the Committee with a sincere vote of thanks and an allotment of funds to complete its work and close its affairs.

In addition to those signatory to this report the following have been members of the Committee: Messrs. Julian Hinds, Robert A. Monroe, A. L. Sonderegger, William Franklin Allison (deceased), Samuel Fortier (deceased), and Stuart Sims.

In the $11\frac{1}{2}$ years of its existence, the Committee has held 17 meetings, at which the average attendance has been more than 80 per cent.

Plan of Procedure

At its first meeting a plan of procedure was outlined that has been closely followed. At that time the Committee planned:

1.—To select certain major subjects on which the Committee's activities would be concentrated.

2.—To prepare first a complete bibliography covering those subjects.

[1] Presented at the Annual Meeting, New York, N. Y., January 17, 1934.

3.—To appoint as co-operating members engineers who have had special training and opportunities for study and research in each of the subjects selected.

4.—To stimulate the preparation of papers covering these subjects by those having special knowledge and competency in their preparation.

5.—To bring to light much valuable data and information in annual and special reports in the files of Federal and private agencies, that otherwise would be lost to the Engineering Profession.

6.—To stimulate original research by those having opportunities and to aid in securing funds therefor.

7.—To arrange for the installation of suitable apparatus in new structures to test their behavior after being placed in operation, such as piezometer pipes in siphon spillways, pressure pipes in dams, facilities for testing flow in lateral flow spillways, etc.

8.—To test fundamental theories by laboratory experiments enlisting the aid of facilities and operators in the hydraulic laboratories of engineering colleges and elsewhere.

9.—To co-operate with all agencies doing work of a similar nature, and to avoid duplication of efforts among groups of the Society.

Subjects Selected.—About twenty subjects were suggested and considered, of which the following were first selected by vote as being most worthy of study: (I) Evaporation from Reservoirs; (II) Canal Conversions; (III) Water Movement and Pressure Under Dams; (IV) Siphon Spillways; (V) The Silt Problem; (VI) Chutes and Drops; (VII) Scouring Below Dams; (VIII) Irrigation Deliveries; (IX) Lateral Flow Spillways; and (X) Permissible Canal Velocities. Subsequently, the following were added: (I-*A*) Evaporation from Soils; and (XI) Fishways and Fish Screens.

The Committee's work could not be restricted to irrigation alone, as nearly every subject listed is just as important in other lines of hydraulic engineering.

In addition to securing data on each of the subjects listed, the Committee prepared a complete bibliography in each subject. It has also prepared a glossary of terms and a set of standard symbols for writers in these subjects.

The tangible evidence of the Committee's activities is contained in the papers it has secured and presented to the Engineering Profession through publications of the Society and others. These are summarized briefly as follows:

Bibliography.—All important publications were searched for pertinent papers, resulting in a comprehensive bibliography. This bibliography (*Proceedings*, Am. Soc. C. E., March, 1925, Society Affairs, p. 147), gives the reference and a synopsis of the article.

Standard Symbols and Glossary of Terms Used in Hydraulics as Applied to Irrigation.—The Committee has labored about five years in the preparation of a list of standard symbols and a glossary of terms used in hydraulics as applied to irrigation. The preliminary work was published in *Proceedings*,

Am. Soc. C. E., May, 1932, p. 729. The discussion which followed was voluminous. It was not published, but was transmitted to the Committee. These comments were taken seriously and most of the definitions were revised in conformity with the information and suggestions supplied. Since then, mimeographed copies of the revised glossary have been sent to about 200 hydraulicians with a request for additional suggestions. It is expected that the return from these requests will result in a final draft for publication as a manual, or in other suitable form, by the Society.

(I) and (I-A).—Evaporation from Reservoir and Soils.—The Committee's work in these subjects was completed in 1932, and by agreement further consideration was relinquished to the Irrigation Division. One important objective the Committee had was to determine a set of standard coefficients by which records of evaporation from a large variety of pans could be converted to that from a large water surface. To this end a copper-lined reservoir 84 ft in diameter was constructed at Fort Collins, Colo., by the United States Bureau of Agricultural Engineering, in co-operation with the Colorado Experiment Station. Elaborate observations were made for a period of several years. The final paper—A Symposium on Evaporation from Water Surfaces—contains a summary of most of the existing evaporation data.

The papers secured on these subjects through the Committee's efforts are:

(1) "Evaporation on United States Reclamation Projects," by Ivan E. Houk, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 90 (1927), p. 266.

(2) "Experiments to Determine Rate of Evaporation from Saturated Soils and River-Bed Sands," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 94 (1930), p. 961.

(3) "Water Supply from Rainfall on Valley Floors," by A. L. Sonderegger, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 94 (1930), p. 1242.

(4) "Evaporation as a Function of Insolation," by Burt Richardson, Esq., *Transactions,* Am. Soc. C. E., Vol. 95 (1931) p. 996.

(5) Evaporation from Water Surfaces: A Symposium: "Evaporation from Different Types of Pans," by Carl Rohwer, Assoc. M. Am. Soc. C. E.; "Evaporation from Reservoir Surfaces." by Robert Follansbee, M. Am. Soc. C. E.; and "Standard Equipment for Evaporation Stations," Final Report of Sub-Committee on Evaporation of the Special Committee on Irrigation Hydraulics, *Proceedings,* Am. Soc. C. E., February, 1933, p. 221 *et seq.*

(II).—Canal Conversions.—A large mass of data scattered through the files of the United States Bureau of Reclamation was assembled and supplemented by field measurements. Laboratory experiments have also been made. The assemblage of data appearing as a result of the Committee's efforts includes:

(1) "The Hydraulic Design of Flume and Siphon Transitions," by Julian Hinds, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 92 (1928), p. 1423.

(2) "Determining the Energy Loss in the Hydraulic Jump," by J. C. Stevens, M. Am. Soc. C. E., *Engineering News-Record,* July 22, 1926.

(3) "The Hydraulic Jump in Standard Conduits," by J. C. Stevens, M. Am. Soc. C. E., *Civil Engineering,* October, 1933, p. 565.

One of the perplexing problems before hydraulicians is energy losses in expanding conduits. The Committee has secured considerable data without, however, arriving at any definite conclusions. Among these data are the results of many experiments undertaken largely at the instance of the Committee, as follows:

(4) "Experiments on the Flow Through Flume Transitions": A Thesis, by Philip F. Thayer and J. Perry Yates, Juniors, Am. Soc. C. E., University of California, Berkeley, Calif., prepared as a result of laboratory experiments. A synopsis of this Thesis appears in *Proceedings,* Am. Soc. C. E., March, 1927, Society Affairs, p. 127.

(5) "Loss of Head in Expanding Conduits": A Thesis, by Messrs. Samuel Finlay and Jorge Altimirano, University of Santiago, Chile. This Thesis gives the results of about 830 experiments on flow through expanding flumes of varying characteristics. The basic data for these experiments were recomputed by J. C. Stevens, M. Am. Soc. C. E., and somewhat more consistent results obtained. This review is in the form of a paper in the Committee's files.

(6) "Experiments on Expanding Section," by Ralph Parshall, Assoc. M. Am. Soc. C. E., a paper prepared as a result of experiments at Bellvue Laboratory, Fort Collins, Colo.

(7) "Open Channel Expansion": A Thesis, by D. D. Curtis, Assoc. M. Am. Soc. C. E., prepared as a result of a series of experiments made in 1931 at the Hydraulic Laboratory of the University of Iowa, Iowa City, Iowa.

(III).—Water Movement and Pressure Under Dams. The Committee has been responsible in part for the systematic installation of pipes in dams both recently completed and now under construction for determining uplift pressures, not only at bed contacts with the foundation rock, but also in horizontal construction joints. Certain laboratory experiments have been made and others are in progress. Five important papers are available as a result of the Committee's work:

(1) "Upward Pressure Under Dams; Experiments by the United States Bureau of Reclamation," by Julian Hinds, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 93 (1929), p. 1527.

(2) "Water Uplift in Construction Joints of Concrete Dams": A Thesis, by the late John E. Skafte, Jun. Am. Soc. C. E., California Institute of Technology, Pasadena, Calif.

(3) "Uplift Pressure Measurements at Masonry Dams," by Ivan E. Houk, M. Am. Soc. C. E., *Civil Engineering,* September, 1932, p. 578.

Using the material gathered, D. C. Henny, M. Am. Soc. C. E., at the request of the Committee, prepared the following paper. It presents the theory of uplift in an entirely new light, and develops a rational method of treating this most important factor in the design of dams:

(4) "Stability of Straight Concrete Gravity Dams," by D. C. Henny, M. Am. Soc. C. E., *Proceedings,* Am. Soc. C. E., September, 1933, p. 1071.

(IV).—Siphon Spillways.—The Committee has induced the builders of several recently constructed siphons to install a series of piezometers so that their behavior may be tested. These include: Fuller Lake Siphon, Nevada Irrigation District, California; Dwinnel Reservoir Siphon, Montague Water Conservation District, California; Tiger Creek Conduit Siphon, Pacific Gas and Electric Company, Mokelumne River, California; and Leaburg Siphons, McKenzie River, City of Eugene, Oregon.

In addition, much laboratory experimental work has been done. The following papers contain the essential data secured by the Committee:

(1) A translation and recomputation of the losses given by E. Scimeni in his paper, "Sul Rendimento dei Sifoni Autolivellatari," *Electrotechnica,* May 5, 1928. (In Committee's files.)

(2) "Experiments on a Model Siphon": A Thesis, by Messrs Brunner Gramatky and Kenneth Robinson, California Institute of Technology, Pasadena, Calif.

(3) "The Design and Operation of Siphon Spillways," a Thesis, by Jason Plowe, University of California, Berkeley, Calif.

(4) "Siphon-Spillway Models Tested against Prototypes," by Herbert H. Wheaton, Assoc. M. Am. Soc. C. E., *Engineering News-Record,* August 18, 1932.

(5) "On the Behavior of Siphons," by J. C. Stevens, M. Am. Soc. C. E., *Proceedings,* Am. Soc. C. E., August, 1933, p. 925.

(V).—The Silt Problem.—Many data have been gathered on the silt content of streams, the silting of reservoirs, and the control of silt in canals and reservoirs. The Committee's Final Report has been prepared by Mr. Stevens at the request of the Committee, the Silt Committee of the Power Division, and the Executive Committee of the Irrigation Division, as follows:

"The Silt Problem," by J. C. Stevens, M. Am. Soc. C. E. (Manuscript has been transmitted to the Society's Secretary for publication.)

In addition the following have appeared in the Society's publications largely as a result of the Committee's efforts:

(1) "Report on the Silt Problem," by Fred D. Pyle and Franklin Thomas, Members, Am. Soc. C. E., with additions by the Committee, *Proceedings,* Am. Soc. C. E., March, 1925, Society Affairs, p. 141.

(2) "Silting of Reservoirs," with Bibliography, by Kirk Bryan, Esq., *Proceedings,* Am. Soc. C. E., March, 1927, Society Affairs, p. 129.

(3) "Sand Control Works at Fort Laramie Canal Intake," by Ivan E. Houk, M. Am. Soc. C. E., *Engineering News-Record,* June 14, 1928.

(4) "Sand Problems at Franklin Canal Intake, Rio Grande Project," by Ivan E. Houk, M. Am. Soc. C. E., *Western Construction News,* November 25, 1928.

(5) The status of the silting of Elephant Butte and McMillan Reservoirs is set forth by R. F. Walter, M. Am. Soc. C. E. in discussion of the paper,

"Silting of Lake Austin," by T. U. Taylor, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1719.

(VI).—Chutes and Drops.—An attempt was made to have field tests made on the hydraulic behavior of existing structures. Tentative promises of appropriations from the Division of Agricultural Engineering, U. S. Department of Agriculture and the U. S. Bureau of Reclamation were made, but owing to other demands these were never realized and the Committee postponed further consideration of this subject until some co-operative arrangements, with the necessary funds, could be secured. In its place the subject of "Fishways" was taken up.

(VII).—Scouring Below Dams.—The Committee's first endeavor was to secure a summary of existing information on this subject. The results appear as a Thesis on "The Scouring Effect of Water Below Dams," by I. Neudatchin (name since changed to Ivan M. Nelidov), University of California, Berkeley, Calif., a synopsis of which appears in *Proceedings*, Am. Soc. C. E., March, 1927, Society Affairs, p. 125.

Perhaps the most comprehensive collection of data on this subject in existence has been brought out through the Committee's efforts in the following paper and the discussions prompted by it:

(1) "Baffle-Pier Experiments on Models of Pit River Dams," by I. C. Steele and R. A. Monroe, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 451.

(VIII).—Irrigation Deliveries.—The necessity for suitable devices for measuring irrigation water in any volume under all conditions of silting, aquatic plant growth, and where heads for weirs or other devices were not available, is apparent. With the Committee's aid and encouragement the Parshall Measuring Flume (formerly called the "Improved Venturi Flume") has been developed. It was in the making when the Committee was organized. Mr. V. M. Cone made the first Venturi flume and from his idea has been evolved the present Parshall measuring flume as a result of co-operative efforts between the U. S. Bureau of Agricultural Engineering and the Colorado Experiment Station, in which R. L. Parshall, Assoc. M. Am. Soc. C. E., was most active. There are Parshall flumes of many sizes now in service, measuring discharges from 1 to 3 000 sec-ft.

Another important phase of this subject is the quantity of irrigation water actually delivered to farm lands. The following papers have appeared at the request of the Committee:

(1) "The Improved Venturi Flume" (now called the Parshall Measuring Flume), by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 841. The data in this paper with some additions and revisions are contained in *Bulletin 336*, Colorado Experiment Station, Fort Collins, Colo., under the same title and authorship.

(2) "Use of Water on Federal Irrigation Projects," by E. B. Debler, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1195.

(3) "Measuring Irrigation Deliveries in the Panjab," by E. S. Lindley, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1005.

(IX).—Lateral Flow Spillways.—Spillways of this character are becoming of increasing importance. Two important papers have been secured by the Committee as follows:

(1) "Side Channel Spillways; Hydraulic Theory, Economic Factors, and Experimental Determination of Losses," by Julian Hinds, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 881.

(2) "Side Spillways for Regulating Diversion Canals," by W. H. R. Nimmo, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1561.

Mr. Hinds' analysis is predicated on the hypothesis that the energy of flow over the spillway crest is entirely lost at right angles thereto in the direction of outflow. In order to develop this phase of the problem the Committee secured the following papers:

(3) "Theoretical Energy Losses in Intersecting Pipes," by J. C. Stevens, M. Am. Soc. C. E., *Engineering News-Record*, June 4, 1925.

(4) "Experiments on the Losses in Intersecting Pipes": A Thesis, by Lawrence P. Sowles, Jun. Am. Soc. C. E., and M. Bernard McGowan, Assoc. M. Am. Soc. C. E., College of Civil Engineering, University of California, Berkeley, Calif.

(5) "A Study of Impact Effect:" A Thesis by Messrs. S. Gale Herrick and J. Frank Jorgensen, University of California, Berkeley, Calif.

(X).—Permissible Canal Velocities.—The Committee's first activity was to send out a questionnaire to irrigation managers, engineers, and others competent to speak with authority on this subject. The data thus secured, together with other data in hand, were presented in a final report on this subject, which appeared as:

(1) "Permissible Canal Velocities," by the late Samuel Fortier and Fred C. Scobey, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 940.

(XI).—Fishways and Fish Screens.—This subject is becoming increasingly important to the West, especially on those streams where migratory fish are caught commercially. The present status of existing knowledge on this subject is well presented in a paper recently secured by the Committee, as follows:

(1) "Problems of Fishway Construction," by Shirley Baker, M. Am. Soc. C. E., and U. B. Gilroy, Assoc. M. Am. Soc. C. E., *Civil Engineering*, December, 1933, p. 671.

In conclusion it should be remembered that the Committee has had no funds except for the traveling expenses of its members for their attendance at meetings and for clerical expenses of the Secretary's office. The expenses of field and laboratory work were paid entirely by other agencies that were induced to undertake work for the Committee.

The single task of having unearthed many valuable data on the several subjects under investigation, that would otherwise have remained buried in the files of the U. S. Bureau of Reclamation would itself have fully justified the existence of this Committee. The presentation of the data secured through papers under the names of individuals is believed to be a sound policy. It stimulates research and authorship, places credit where justly due, and elicits discussion that ofttimes is fully as valuable as the paper itself; but more important than all, it places in permanent form for reference, data that otherwise would ultimately be quite lost to the profession were the Committee's productions limited by its progress reports.

Respectfully submitted,

J. C. STEVENS,
Secretary.

November 15, 1933.

Special Committee on Irrigation Hydraulics:

D. C. HENNY, *Chairman,*
J. C. STEVENS, *Secretary,*
B. A. ETCHEVERRY,
J. L. SAVAGE,
FRED C. SCOBEY,
FRANKLIN THOMAS,
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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

EVAPORATION FROM WATER SURFACES A SYMPOSIUM

Discussion

BY MESSRS. CARL ROHWER, ROBERT FOLLANSBEE, AND THE SUB-COMMITTEE ON EVAPORATION, SPECIAL COMMITTEE ON IRRIGATION HYDRAULICS

CARL ROHWER,[46] ASSOC. M. AM. SOC. C. E. (by letter)[46a].—In preparing this paper the writer had no intention to minimize the importance of the work being done by Messrs. Richardson, Cummings, and McEwen on the relation between insolation and evaporation. It is generally conceded that, over a period of time, the quantity of water evaporated will depend on the insolation. There are several practical difficulties, however, which must be overcome before this method can be used for the determination of the evaporation losses from water surfaces and, until more information is available, the method cannot be used generally in evaluating the evaporation loss from a reservoir.

The mere fact that the evaporation data do not plot consistently when adjusted to a fixed value of one of the factors affecting evaporation, does not necessarily mean that the original data, or that the pan coefficients, are incorrect, as suggested by Mr. Randell. The apparent inconsistency in the data may be due to several causes: First, the method of adjusting them to a fixed value of one of the factors affecting the result may be in error; second, the meteorological data given may not have been taken at the evaporation pan; and, third, the meteorological data, although taken at the evaporation pan, may have been observed at such times that the means do not represent the true conditions. Any one or all three factors might cause inconsistencies in the plotted results.

The writer agrees with the statements of Mr. Grunsky concerning the evaporation from the U. S. Weather Bureau, Class *A* land pan. He admits

NOTE.—This Symposium on Evaporation from Water Surfaces was published in February, 1933, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings* as follows: May, 1933, by Messrs. Ralph R. Randell, C. E. Grunsky, and Charles H. Lee; August, 1933, by Messrs. J. T. Harding, L. Standish Hall, and Adolpho Santos, Jr.; November, 1933, by John W. Pritchett, Assoc. M. Am. Soc. C. E.; and December, 1933, by Messrs. Ivan E. Houk, and R. I. Meeker.

[46] Associate Irrig. Engr., Colorado Experiment Station, Fort Collins, Colo.

[46a] Received by the Secretary February 3, 1934.

also that the evaporation from a sunken circular pan provided with a shield to protect the exposed rim from sunshine would approach more nearly the evaporation from a large body of water. Records from the latter type of pan are not available, however. Although the evaporation from the Weather Bureau pan is much greater than that from a lake or reservoir, this type of pan has many advantages, as has been stated, and for these reasons it is recommended for use.

There is a noticeable lack of uniformity in the method of maintaining floating pans, which is, in part, the cause of the inconsistent results obtained from these pans. The data as to the setting of the floating pan for which evaporation records were reported by the writer, were given to acquaint the reader with the conditions under which the readings were taken and not with the intention of recommending the particular setting. As pointed out by the writer, the water in the U. S. Geological Survey floating pan should be maintained at the same level inside as outside.

Mr. Pritchett calls attention to additional records on evaporation from pans of different sizes and states that the reservoir at Austin, Tex., is 30 by 200 ft, instead of 20 by 150 ft. It is the writer's understanding that the size of the reservoir varies with the rainfall and is at times even smaller than 20 by 150 ft.

The writer believes that the suggestion made by Mr. Lee in reference to changing the designation of the Type 2 pan to Type 3, in Table 5(*f*), should be followed, and that consideration should be given to the fact, as he points out, that the Type 3 pan was not exposed under standard conditions.

The evaporation records submitted by Mr. Hall provide additional information as to the relation between the evaporation from floating pans and from other types. It is unfortunate that this information was not included in the writer's paper, as only a few records concerning the comparison between the evaporation from floating pans and land pans were obtained. The writer does not agree, however, with Mr. Hall's recommendations as to the types of pans to be used for the study of the evaporation from reservoirs. His own records (Table 13) show that the results obtained from the floating pans are more erratic than those from land pans.

The observations by the writer on the floating pan in the 85-ft reservoir at Fort Collins, Colo., were made under laboratory conditions where it was possible to measure the evaporation accurately and to observe whether water was splashing into or out of the pan. It is doubtful whether as consistent results would be obtained under field conditions. The land pans at Stonyford, Calif., were installed in damp sand, but the conditions were scarcely similar to those under which floating pans are exposed.

The ratios given by the writer in Table 6 are based on the available records, but they are not final. Whenever additional information on the evaporation from different types of pans becomes available the ratios given should be recomputed on the basis of all the data.

Mr. Houk calls attention to the effect of variation in the humidity near large bodies of water on the evaporation from pans. It is generally accepted

that the air is damper near lakes and reservoirs; but the effect of the water on the evaporation is not as great as might be expected, for two reasons: (1) Because moist air is lighter than dry air, the moist air is rising continuously and being replaced by drier air (but for this fact and the influence of the wind, the air near a lake or reservoir would soon become saturated); and (2) the rate of evaporation is proportional to the difference in vapor pressure between the water surface and the air above it. In arid sections (particularly in the summer), the vapor pressure of the water surface is much greater than the vapor pressure of the air, and, consequently, small changes in the vapor pressure of the air due to proximity to a large body of water have a relatively small effect on the rate of evaporation of pans near the lake or reservoir. This conclusion is supported by the observations in Table 6, Items Nos. 44 to 49, inclusive, which show that the correction factors for obtaining the reservoir equivalent are practically the same whether determined near a lake or away from the influence of a large body of water.

The observations reported by Professor Harding pertaining to the evaporation from small land and floating pans located on the leeward and windward sides of the 12-ft evaporation pan at Escalante, Utah, indicate that the evaporation is less on the lee side of the large pan. This may be due to the difference in humidity, but other factors, such as wind and temperature, may have some effect also. The effect of wind is clearly shown by Professor Harding by the observations on the pan at the Fallon Experiment Station. Wind is more likely to be the cause of differences in evaporation records than temperature or humidity. For this reason the pan should be exposed as nearly as possible to the same wind conditions as the reservoir for which the rate of evaporation is desired.

In conclusion, the writer would like to call attention to the fact that the ratios given in his paper are averages, usually covering several months, and are based on the total evaporation for the period and not on the average of the monthly ratios. For this reason the ratio for a particular month of the year may differ from the average for the period. This fact should be given consideration when computing monthly rates of evaporation, particularly for the winter months when the pan may be frozen while the reservoir still remains open.

ROBERT FOLLANSBEE,[47] M. AM. SOC. C. E. (by letter)[47a].—In publishing all available records of evaporation (the real purpose of the writer's paper) two courses were open: (1) To present the records as observed with the knowledge that, obviously, they were not comparable, owing to the various methods by which they were obtained, and thus could not give even an approximately true picture of the variation in evaporation due to variations in meteorological factors; or (2) with the aid of known comparisons to reduce all records to approximate reservoir surface evaporation which would be much more nearly comparable than the observed records and, at the same time, would present the factor used in each case, in order that the user of the records might

[47] Dist. Engr., U. S. Geological Survey, Denver, Colo.

[47a] Received by the Secretary December 18, 1933.

easily deduce the originals and thus correct them by any factor he chose, based on additional information. The latter method was selected.

In selecting the reservoir surface method the writer was well aware of the fact that in some instances the known reduction factors were probably subject to error, and that the resulting values would appear inconsistent when compared with the accompanying meteorological factors. One reason for these inconsistencies undoubtedly lies in the fact that these factors are not in every instance directly comparable. While an attempt was made to obtain exact information regarding them and the points at which they were observed, such attempt was not always successful. Particularly is this true of the foreign records presented in this closure. One of the greatest uncertainties is the mean value used for relative humidity. At only a very few stations are continuous records of relative humidity available, and the great majority rely on two or three readings per day, taken during the daylight hours. In the case of the Australian records (Table 17 (*g*)) one reading per day is taken. In some climates the error in computing the mean daily relative humidity from one, two, or (at regular U. S. Weather Bureau stations), three, readings per day may give the true daily mean within a few per cent., but in other climates considerable error may be introduced. Mr. Pritchett's discussion of the writer's comparison between the evaporation at Washington, D. C., and Austin, Tex., as given in Table 10, illustrates the variation in relative humidity at different hours of the day. Mr. Pritchett states that the relative humidity at Austin, as shown by a 14-yr record, is 82%, whereas the writer had given 68% in his record. From subsequent correspondence, it has been learned that the value of 82% was based on one reading per day at 8 A. M., whereas the Meteorological Observer at the University of Texas states that the mean for the day is about 66%, the method of arriving at the mean not being given. This value agrees closely with the 68% used by the writer.

Another uncertainty is the height above the ground at which the wind velocity is measured, and the exact exposure of the evaporation pan as compared with the point at which the wind velocity is measured. As an additional example of the effect of wind on the rate of evaporation two records at Singapore may be cited. The Water Engineer's Department maintains two similar evaporation pans not very far apart, the mean annual evaporation for the same period being 52.95 in. for one and 42.33 in. for the other. The explanation for this difference is that one station is less exposed to the wind than the other.

The monthly records of evaporation which are deposited in Engineering Societies Library, in New York, N Y., give all available information regarding the various factors, but it was obviously impossible to present the details in the yearly summaries published in the writer's paper. The monthly records also give the source of each record.

Another reason for apparent discrepancies in some instances probably may be charged to human fallibility, and lies in errors in the observed records

themselves; but, undoubtedly, there are errors in the reduction factors used, as they probably do not apply strictly in every instance.

Even with the known limitations it was felt that the bringing together on as comparable a basis as possible (and that basis the one that the engineer needs in his use of evaporation records), all records known to the writer, was a worthwhile contribution to the subject of evaporation from the practical engineering standpoint.

Mr. Grunsky's criticism of the floating pan as a method of measuring evaporation is corroborated by the discrepant comparisons cited by the writer in his original paper. Two other comparisons have come to the writer's attention. In Turkestan, E. M. Oldekop, a Russian engineer, compared records of evaporation between floating pans and land pans, and came to the conclusion that a factor of 0.73 was required to reduce the records of the former to reservoir surface evaporation.[48] Near Paramaribo, Suriname (Dutch Guiana), Dutch engineers compared the evaporation between a circular tank of concrete, 5.25 ft in diameter and 2.5 ft deep, sunk in the ground and three concrete tanks which have a rectangular surface, 3.3 by 2.6 ft, suspended in the water at a distance of 2 miles from the ground tank, and found that the floating tanks gave results which were 89% of those of the ground tank. Assuming that the ground tank requires a coefficient of 0.95 to reduce it to reservoir surface evaporation the comparison indicates that the coefficient for the floating tanks is 1.06.[49] Table 17, Item 9(*a*) is a circular concrete pan, 6.3 ft in diameter and 2.95 ft deep, sunk in the ground, with the top 4 in. above the surface and a coefficient of 0.95. As far as the writer has been able to learn, practically no attempts have been made in foreign countries to compare evaporation records from pans with those of larger bodies of water, and Sleight's classic experiments at Denver, Colo., are still the nearest approach to it that have been attempted.

From what has been stated, it is not surprising that Mr. Randell found discrepancies in the records presented by the writer, and it is obviously impossible to reduce them to one standard of meteorological factors, as much as such a procedure might be desirable. Other commentators have called attention to variations in these factors within a comparatively short distance of the evaporation pans, thus further emphasizing the fact that they are not directly comparable in many instances. The various comments are most valuable in bringing out the fact that evaporation records, even when carefully taken, are subject to considerable error and cannot be accepted as exact.

It may be true, as suggested by Mr. Santos, that temperature, relative humidity, and wind velocity are not the only factors influencing evaporation, but in the vast majority of the existing records these are the only factors known and, in many instances, they are known only imperfectly.

Mr. Houk calls attention to the fact, recognized by the writer, that in arid regions, where large reservoirs are most likely to be constructed, relative humidity of the atmosphere adjacent to the water surface will be considerably

[48] "The Evaporation from the Surface of the River Syr-Darya According to Observations at the Station Saporozskaya," *Journal of Irrigation No. 1,* 1924, Tashkent.

[49] *De Waterstaats Ingenieur.* May, 1924, No. 5, Batavia, Java.

TABLE 17.—Summary of Evaporation Records Reduced to Reservoir Surface Evaporation

Item	Station	Elevation, in feet	Years	Temperature of air, in degrees Fahrenheit	Wind velocity, in miles per hour	Relative humidity (percentage)	Reservoir Surface Evaporation, in Inches		Coefficient for pan
							April-September*	Annual	
	(a) Mexico								
1	Tacubaya (City of Mexico)	7 575	10	58	5.6	62	21.71	39.35	0.69
2	Presa Calles, Aguascalientes	6 644	10	62	..	..	29.31	49.81	0.69
3	Guadalajara, Jalisco	5 194	10	66	3.8	57	37.79	64.85	0.69
4	San Buenaventura, Chihuahua	5 164	10	62	..	..	37.26	58.31	0.69
5	Jalapa, Vera Cruz	4 590	10	64	2.5	78	20.73	37.93	0.69
6	Concordia, Coahuila	3 625	10	69	..	..	47.18	72.76	0.69
7	Don Martin, Coahuila	787	10	..	..	..	52.20	74.42	0.69
8	Santa Rosalia, Tamaulipas	.....	10	72	..	..	46.02	67.36	0.69
	(b) South America								
9	Georgetown, British Guiana	.....	1916–31	80	3.2	80	27.96	55.94	0.95
9 (a)	Paramaribo, Dutch Guiana	.....	1	80	2.0	83	24.21	49.03	0.95
	(c) West Indies								
10	Kingston, Jamaica	100	1924–30	79	7.6	79	33.07	55.54	0.75
	(d) Europe								
11	Lake Hjalmaren, Sweden[2]	315	1889–1929	42	10.2	80	12.58	14.98	0.83
12	Talla Water, Scotland	904	7	....	..	..	11.47	13.33	0.95
13	Glencorse, Scotland	639	1862–1904	47	..	81	11.44	13.97	0.95
14	Reevsby, Lincoln, England	124	6	....	..	..	10.16	13.68	0.95
15	Southport, Lancashire, England	37	7	49	..	82	10.70	12.23	0.80
16	Ormsby, St. Michael, Norfolk, England	21	7	48	..	..	13.46	16.56	0.95
17	Lower Laithe, Yorkshire, England	772	7	....	..	..	11.87	13.08	0.95
18	Ardsley, 1, Yorkshire, England	395	7	....	..	..	13.66	16.09	0.95
19	Ardsley, 2, Yorkshire, England	326	7	....	..	..	12.29	14.14	0.95
20	Harrowgate, Yorkshire, England	589	7	48	..	..	14.28	18.15	0.95
21	Petersfield, Hampshire, England	747	7	51	..	..	10.99	13.24	0.83
22	Otterbourne, Hampshire, England	112	7	50	10.7	..	15.36	18.22	0.95
23	Derwent Dam, Derbyshire, England	.....	1906–12	....	..	..	10.44	13.08	0.95
24	Camden Square, London, England	111	1885–1931	50	0.5	79	12.84	14.73	0.95
25	Lee Bridge, London, England	.....	1860–73	50	..	..	13.20	17.18	0.83
26	Kennick, South Devon, England	835	7	....	..	..	15.17	17.72	0.95
27	Plymouth, England	755	1907–31	49	..	..	12.42	16.10	0.95
28	Emdrup, Denmark[3]	.....	1849–59	45	..	..	19.34	23.17	0.83
29	De bilt, Holland[3]	10	1910–27	48	9.0	81	21.32	26.83	0.83
30	Agerisee, Switzerland	2 352	1912	51	..	..	18.71	29.15	1.00
31	Zugersee, Switzerland	1 368	1912	53	..	78	19.46	30.49	1.00
32	Grimnetzsee, Germany	.....	1909–13	48	5.5	78	24.10	29.16	0.83
33	Marathon Reservoir, Greece	830	1926–32	63	4.4	63	47.57	59.51	0.75
34	Marathon Reservoir, Greece	728	1926–32	63	4.4	63	48.11	62.72	0.83
	(e) Africa								
35	Cleveland Dam, South Rhodesia	.....	1913–31	65	..	60	39.41	71.04	0.83
36	Kopje Alleen, Transvaal	3 488	1926–27	66	..	61	44.83	72.75	0.95
37	Hartebeest, Poort, Transvaal	4 000	1926–30	66	..	63	42.83	67.51	0.95
38	Johannesburg, Orange Free State	5 925	....	60	12.5	66	35.12	62.36	0.95
39	Grootfontein, Cape Province	4 130	1926–27	62	..	59	50.77	77.93	0.95
40	Grassridge Dam, Cape Province	3 500	1926–30	62	..	65	51.97	76.92	0.95
41	Lake Arthur Reservoir, Cape Province	4 000	1926–30	....	..	..	47.93	71.73	0.95
42	Graaff Reinet, Cape Province	2 463	1926–30	65	..	59	53.29	79.94	0.95
43	Kamanassie Dam, Cape Province	1 100	1926–30	66	..	71	50.02	71.75	0.95
44	Lake Mentz, Cape Province	800	1926–30	67	..	69	48.42	68.26	0.95
45	Steenbras Reservoir, Cape Province	1 130	1922–31	59	..	66	39.83	53.80	0.95
46	Molteno Reservoir, Cape Province	310	1922–31	....	..	..	40.29	54.10	0.85
47	Windhoek, South West Africa	5 676	1928–31	66	7.6	30	57.07	94.95	0.94

* For stations in Southern Hemisphere, period is October to March.

TABLE 17.—(*Continued*)

Item	Station	Elevation, in feet	Years	Temperature of air, in degrees Fahrenheit	Wind velocity, in miles per hour	Relative humidity (percentage)	Reservoir Surface Evaporation, in Inches		Coefficient for pan
							April-September*	Annual	
	(f) Asia and Islands								
48	Saporozskaya, Turkestan	995	1912–17	58	10.0	55	.....	54.35	0.83
49	Kerki, Turkestan	879	1911–17	62	..	61	38.69	53.14	0.83
50	Tsining, Shantung, China	.....	1915–18	....	..	..	40.86	56.94	0.83
51	Shasi, Hupeh, China	.....	1929	64	..	..	23.25	30.96	0.83
52	Tuanfeng, Hupeh, China	.....	1929–30	64	..	..	23.96	33.43	0.83
53	Yukwanshan, Hupeh, China	82	1929–30	64	..	..	13.04	20.17	0.83
54	Anking, Anhwei, China	40	1929	67	..	..	24.73	35.69	0.83
55	Nanchang, Kiangsi, China	.....	1929	65	..	..	26.46	39.32	0.83
56	Kiukiang, Kiangsi, China	65	1929–30	62	..	..	25.51	35.79	0.83
57	Siangyin, Hunan, China	.....	1929	66	..	..	18.59	26.77	0.83
58	Singapore, Straits Settlement	.....	1927–31	81	..	81	26.34	52.95	0.75
59	Wadoek, Soembersono, Java	197	1914–23	79	..	80	27.34	53.39	0.83
60	Seoul, Korea	.....	1907–26	52	4.2	70	23.36	32.51	0.75
61	Sapporo, Hokkaido, Japan[1]	.....	1907–26	44	7.6	79	21.84	30.73	0.75
62	Osaka, Japan[1]	.....	1907–26	59	7.5	74	26.66	39.32	0.75
63	Tokyo, Japan[1]	.....	1907–26	57	6.0	74	18.51	29.34	0.75
64	Tainan, Formosa	.....	1907–26	73	7.1	80	28.92	50.36	0.75
65	Tansa Lake, Bombay, India	300	1906–21	81	..	78	24.08	50.71	0.85
66	Khadakwasla, India	.....	1924–32	....	..	..	27.10	56.41	0.83
67	Bhatgar, India	319	1917–32	....	..	..	29.35	65.49	0.88
	Mettur Reservoir, Madras, India	.....	1	84	4.9	82	38.48	72.18	0.69
	(g) Australia								
68	Rockhampton, Queensland	37	14	73	..	67	28.05	46.96	0.90
69	Brisbane, Queensland	137	22	69	6.1	68	32.47	50.37	0.90
70	Cataract River, New South Wales	520	....	....	..	..	20.64	29.07	0.82
71	Sydney, New South Wales	138	51	63	8.5	70	22.20	32.01	0.82
72	Bathurst, New South Wales	2 323	16	57	3.2	71	29.11	39.62	0.82
73	Lake George, New South Wales	2 267	27	....	..	..	17.96	25.93	0.90
74	Hay, New South Wales	310	14	63	..	60	31.41	41.02	0.90
75	Leeton, New South Wales	466	16	62	..	60	36.90	47.73	0.90
76	Griffith, New South Wales	420	7	62	..	62	40.69	53.62	0.90
77	Yenda, New South Wales	432	4	62	..	60	40.46	52.88	0.90
78	Walgett, New South Wales	436	19	68	2.4	58	30.33	42.92	0.82
79	Hume, New South Wales	566	7	59	2.5	72	34.46	44.57	0.90
80	Wilcannia, New South Wales	267	25	66	..	52	37.51	52.70	0.82
81	Burrinjuck, New South Wales	1 239	18	58	2.2	70	25.04	32.30	0.90
82	Prospect, New South Wales	203	25	....	..	..	24.96	36.32	0.82
83	Lake Victoria, New South Wales	93	4	62	5.4	62	42.90	58.66	0.90
84	Umberumberka, New South Wales	900	20	66	10.3	58	57.34	80.21	0.90
85	Canberra, New South Wales	1 837	10	56	3.0	69	31.58	41.66	0.90
86	Wyuna, Victoria	323	4	54	..	67	29.78	35.77	0.90
87	Werribee, Victoria	79	19	58	..	72	30.21	41.31	0.90
88	Rutherglen, Victoria	533	19	60	..	66	34.66	43.91	0.90
89	Merbein, Victoria	185	10	62	..	66	46.21	63.15	0.90
90	Melbourne, Victoria	115	58	57	8.8	68	26.36	35.25	0.90
91	Adelaide, South Australia	140	61	63	9.7	55	37.88	49.45	0.90
92	Glen Osmond, South Australia	402	7	61	..	66	39.58	55.40	0.90
93	Alice Springs, South Australia	1 901	37	70	..	37	57.85	86.08	0.90
94	Marble Bar, West Australia	595	15	82	..	40	55.92	89.78	0.90
95	Coolgardie, West Australia	1 389	20	65	..	54	55.57	76.14	0.90
96	Merriden, West Australia	1 046	17	63	..	..	56.08	74.20	0.90
97	Narrogin, West Australia	1 114	15	60	..	..	37.87	50.94	0.90
98	Perth, West Australia	197	28	65	0.4	64	39.36	52.68	0.80
99	Hobart, Tasmania	177	20	54	7.3	68	20.83	28.63	0.90
100	Alexandra, New Zealand	520	1930–32	50	2.9	69	22.67	27.13	0.90

* For stations in Southern Hemisphere, period is October to March.

higher than before such construction, with a corresponding decrease in evaporation. In humid regions, however, the effect on relative humidity will be much less marked. The writer cannot agree with Mr. Houk in thinking that the coefficient of 0.69 for the Yuma Citrus Station is much too high

to show the actual evaporation under existing conditions. As Mr. Houk stated in a previous discussion,[50] the evaporation station known as Yuma Citrus Station is situated on a desert mesa where the (unmeasured) relative humidity must be very much less than that at the Yuma Date Orchard Station situated in an irrigated alfalfa field. The measured wind velocity is nearly twice as great at the Citrus Station as to the Date Orchard Station. Both these conditions are conducive to greater evaporation at the former station. However, if a reservoir were constructed at the former site the relative humidity would be increased and the evaporation decreased, although it appears to the writer that the greater wind velocity would still cause a higher rate of evaporation than in the more sheltered Date Orchard Station.

Evaporation Records from Foreign Countries.—Since transmitting the original paper, correspondence has been carried on with Government officials in every country where evaporation records might be available, and, as a result, nearly 200 foreign records have been obtained, together with records of temperature, wind velocity, and relative humidity, where these were available. These latter records may not be directly comparable in each instance. The records may be divided into three classes: (1) Records taken by means of equipment for which reduction factors to reservoir evaporation are fairly well known: (2) records from the Wild balance, an instrument widely used in foreign countries and for which some comparative results with standard equipment are available; and (3) records from Piche tubes.

The records in Class (1) have been reduced to reservoir evaporation, and the summaries are presented in Table 17. Those for the Wild balance (Class (2)) are presented in Table 18 as observed, but the Piche records (Class (3)), being taken under such widely varying conditions and appearing so discrepant among themselves, are not presented. The monthly records themselves have been deposited in the Engineering Societies Library.

The records of reservoir evaporation in Table 17 are taken by equipment described briefly in Table 19. The three concrete boxes at Tansa Lake in Bombay, India (Table 19, Item (24)), are set at three different levels and remain submerged after the end of the monsoon. They are exposed to the atmosphere in turn as the lake level recedes. At Bhatgar (Table 19, Item (26)), the iron tank is set on a stand in a circular concrete tank, 15 ft in diameter and 7 ft deep. The concrete tank is set 2 ft in the ground. The evaporation records for this station are suspended during the monsoon, June to September, and for that period the evaporation has been estimated. At the Mettur Reservoir (Table 19, Item (27)), records are continuous throughout the year.

In Table 19, Item (29), a type of iron pan is described that rests on the bottom of a second iron pan. The latter is sunk in the ground with a 5-in. space between the sides of the two, which is filled with water. A similar arrangement is reported for Perth, West Australia, where the slate box (Table 19, Item (30)) is placed inside a concrete tank with a clearance of 5 in. on the sides and bottom.

[50] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 282.

The Wild balance consists essentially of a circular pan, 7 in. in diameter, resting on a balance. It is believed that the change in elevation of the pan itself, due to the lessening of the weight through evaporation, is indicated by a pointer on a curved scale.

The following comparisons between records from the Wild balance and other forms of evaporimeters have been noted: M. Audebeau-Bey, a member of the Institute of Egypt, has informed the writer that experiments carried

TABLE 18.—OBSERVED EVAPORATION BY MEANS OF WILD BALANCE IN SHELTER

Item	Station	Elevation, in feet	Years	Temperature of air, in degrees Fahrenheit	Wind velocity, in miles per hour	Relative humidity	Observed Evaporation, in Inches: April–September	Observed Evaporation, in Inches: Annual
	(a) South America							
1	Cordoba, Argentine	......	1910–27	63	5.6	64	*30.80	46.39
	(b) Europe							
2	Bergen, Norway	138	1906–15	45	8.3	78	8.36	14.46
3	Stockholm, Sweden	143	1926–29	42	4.3	77	16.83	21.92
4	Lake Pyhäjärvi, Finland	260	1912	41	9.6	80	18.68	21.64
5	Bukarest, Roumania	......	1891–1905	51	......	67	20.48	26.73
6	Cadiz, Spain	33	1920–26	64	10.3	73	36.32	59.39
	(c) Asia							
7	Saporozskaya, Turkestan	995	1915–17	59	8.9	52	48.34	66.99
8	Kawah Tjiwidei, Java	6 332	1917–18	57	......	91	10.44	20.37
9	Tosari, Java	5 692	1912–18	61	4.8	82	14.67	28.43
10	Tjibodas, Java	4 593	1912–18	64	......	87	12.01	24.19
11	Patjet, Java	3 642	1916–18	66	......	87	12.25	22.90
12	Bandoeng, Java	2 395	1912–18	71	......	81	20.80	40.42
13	Buitenzorg, Java	787	1913–18	77	......	83	20.67	36.25
14	Djember, Java	272	1913–18	77	......	82	24.37	45.04
15	Pekalongan, Java	30	1912–18	79	......	84	15.01	27.24
16	Batavia, Java	26	1912–22	79	2.5	83	11.30	21.72
17	Soerabaja, Java	16	1918–23	79	......	82	25.00	45.49
18	Pasoeroean, Java	16	1914–18	79	3.0	77	29.45	51.94

* Evaporation for period, October to March.

on by him in Egypt during a period of two years in the center of the Nile Delta indicated that records from the Wild balance are about 67% of those from a Piche tube, both instruments being in a shelter with shuttered sides. The later work of the Egyptian Government engineers indicated that the coefficient to reduce Piche records in the same type of shelter to reservoir evaporation was 0.50. This coefficient indicates that the Wild records require a coefficient of 0.75 to reduce them to reservoir evaporation.

The same writer stated that experiments carried on at the Aswan Reservoir justify the conclusion that the records of the Wild balance in a shuttered shelter are much nearer the evaporation from a reservoir surface than those of a Piche tube either in a shuttered shelter or in one with open sides. He did not give any details of those experiments.

At Helwan Observatory, near Cairo, experiments carried on by the Government engineers show the following relations between Piche and Wild records

TABLE 19.—DESCRIPTION OF EQUIPMENT IN TABLE 17

Item (1)	Location (2)	Description (3)	Position (4)	Surface dimensions, in feet (5)	Depth, in inches (6)	Coefficient (7)
1	Mexico	U. S. Weather Bureau Class *A* equipment	....	....	....	0.69
2	Jamaica	Cylindrical copper pan (circular)	Above ground	0.67	4	0.75
3	Japanese Empire	Cylindrical copper pan (circular)	Above ground	0.67	4	0.75
4	Singapore	Cylindrical copper pan (circular)	Above ground	0.67	4	0.75
5	British Guiana	Concrete tank (water maintained 2 ft. deep)	Buried	6 by 6	....	0.95
6	Sweden	Floating pan (circular)	Floating	1.33	....	0.83
7	Great Britain	Galvanized iron pan	Buried	6 by 6	24	0.95
8	Great Britain	Galvanized iron pan	Buried	3 by 3	18	0.80
9	Great Britain	Galvanized iron pan	Buried	4 by 4	21	0.83
10	Great Britain	Slate box*	Floating	3 by 3	12	0.83
11	Denmark	Iron pan, painted white	On a grating in lake	1 by 1	8	0.83
12	Holland	Circular pan	Floating	0.75	....	0.83
13	Switzerland	Direct determination of evaporation by measured inflow and outflow, precipitation, and changes in lake flow. Gains or losses in lake bed believed to be negligible	....	....	....	....
14	Germany	Circular galvanized iron pan	Floating	1.67	20	0.83
15	Greece	Circular galvanized iron pan	Suspended from framework in reservoir	2.0	24	0.83
16	Greece†	Circular galvanized iron pan	Above ground	2.0	24	0.75
17	Union of South Africa	Galvanized iron pan	Buried	6 by 6	24	0.95
18	Union of South Africa	Cleveland Dam pan	Floating	3 by 3	18	0.83
19	Union of South Africa	Molteno Reservoir pan	In concrete embankment	4.5 by 2.5	18	0.85
20	Union of South Africa	Windhoek pan	Buried	6 by 6	18	0.94
21	Turkestan	Pan with area of 1.17 sq ft	Floating	....	....	0.83
22	China	Circular tin pan	Floating on water surface	2.0	12	0.83
23	Java	Concrete box	Suspended between poles in lake	3.3 by 3.3	30	0.83
24	Tansa Lake, Bombay, India	Three concrete boxes	Different levels	4 by 4	18	0.85
25	Khadakwasla, India	Iron tank	Floating	6 by 6	36	0.83
26	Bhatgar, India	Iron tank	On stand in circular concrete tank	6 by 6	36	0.88
27	Mettur Reservoir, Madras, India	U. S. Weather Bureau Class *A* equipment	....	....	....	0.69
28	Australia	Cylindrical iron pan	Buried	3	36	0.82
29	Australia	Cylindrical iron pan resting in another pan		3	36	0.90
30	Perth, West Australia	Slate box within a concrete tank	Buried	3 by 3	36	0.80

* At Lee Bridge, London, England. † Information supplied by R. H. Keays, M. Am. Soc. C. E.

(Piche in single shuttered shelter; Wild in double shuttered shelter): Winter ratio, 1.44; and summer ratio, 1.40.[51] By using the Piche reduction factor of 0.50, the coefficients to the Wild records are 0.72 during the winter months, and 0.70 during the summer months.

At Saporozskaya, Turkestan, direct comparisons between a Wild balance and floating pan have been made for the years 1914-15 and 1916-17.[52] By

[51] "The Nile Basin," Vol. 1, p. 58, Cairo, Egypt, Ministry of Public Works, 1931.

[52] *Journal of Irrigation, No. 1*, 1924, Tashkent.

using 0.83 to reduce the floating-pan records to reservoir surface, the coefficients in Table 20 to reduce the Wild records have been obtained. The Wild balance was apparently placed in a shelter of the shuttered type.

TABLE 20.—COEFFICIENT TO REDUCE RECORDS OBTAINED BY WILD BALANCE

Year	Floating pan × 0.83, in millimeters	Wild balance, in millimeters	Coefficient
1914—15	1 028	1 354	0.76
1916—17	1 192	1 616	0.74

SUB-COMMITTEE ON EVAPORATION[53] (by letter)[53a].—The report of the Sub-Committee did not contain a preamble setting forth the argument leading up to the choice of pans. It was known that a great mass of data on pan evaporation existed. Likewise, it was known that the readings on these pans did not represent the evaporation that could be expected from a large water surface such as that of a reservoir. The most desirable pan to recommend appeared to be one that would co-ordinate with a pan of extensive use in the past and that was not too objectionable for use in the future. From this standpoint the Class *A* Weather Bureau pan was adopted after some discussion among the eleven members of the Special Committee on Irrigation Hydraulics.

The shortcomings of this pan were fully recognized—about in the same terms as set forth by Mr. Grunsky; and still it seemed the best. Although the coefficient is quite far removed from unity, the performance of this pan appeared to be the most consistent, with less chance of some unknown factor materially affecting results than would be the case with other types of pan. Of course, the floating pan with a coefficient of unity would be the best if the apparent readings could be assured as the true readings.

However, it was believed that the observer could never be certain that water had not been gained or lost over the rim during rough water on the surface containing the pan. For most floating pans it was believed that any guard sufficient to prevent wash of water over the rim would also be sufficient to inject a measure of doubt as to the relationship between evaporation in the pan and on the outside surface. It is possible that an outrider of brush, attached to the raft carrying the pan, can be made to ride so low in the water that it will reduce wave action and rocking of the raft and still not affect the wind conditions over the surface of the pan.

The Sub-Committee adopts Mr. Grunsky's specification that the surface of the floating pan shall be maintained below that of the water outside the pan. It also agrees with him that a better pan than the Class *A* pan is desirable. However, it does not believe that the sunken land pan suggested by him should be recommended at this time. The Class *A* pan should be used in all future evaporation station installations until such a time as a

[53] Fred C. Scobey, *Chairman,* Ivan E. Houk, R. L. Parshall, and Carl Rohwer, *Co-Operating Member,* Sub-Committee on Evaporation of the Special Committee on Irrigation Hydraulics.

[53a] Received by the Secretary February 10, 1934.

satisfactory improved type of pan is developed and definitely proved to be better than the Class *A* pan for the purpose of securing long-time accurate records of evaporation.

If such a pan should be developed sometime in the future, it will be necessary to maintain records concurrently on both types of installation until the conversion factor for the two can be determined definitely. Such procedure would be imperative in order that the long-time records of Class *A* evaporation pan could be properly co-ordinated with records secured at the new type.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND STRESSES BY SLOPE DEFLECTION AND CONVERGING APPROXIMATIONS

Discussion

BY JOHN E. GOLDBERG, JUN. AM. SOC. C. E.

JOHN E. GOLDBERG,[16] JUN. AM. SOC. C. E. (by letter)[16a].—The discussions of this paper are encouraging and gratifying to the writer. He cannot but feel that they are, at least, expressions of interest in the methods that he has proposed.

Professor Grinter raises the vital question of time required for an analysis. He presupposes that the writer's method requires so much time as to make it comparatively impracticable for ordinary use. On the contrary, the writer believes that it possesses a definite time advantage over methods heretofore proposed. In February, 1932, for purposes of comparison with the method proposed by the Sub-Committee of the Construction Division on Wind-Bracing in Steel Buildings, the writer applied his method to the lower five stories of the Wilson and Maney twenty-story bent, and completed a solution—including calculation of the deflections of the five stories—in 93 min, which indicates a total time of about 370 min for the complete bent. The analysis of the bent by the Sub-Committee's method required about 800 min for moments alone, plus an additional $2\frac{1}{2}$ days for calculation of deflections along two lines of columns, or about 1 800 min for a complete solution, inasmuch as, by any method but a deflection method, it is advisable to calculate deflections along two lines of columns as a check on the accuracy of the solution. At the same time, the writer's solution gave values consistently more accurate than that of the Sub-Committee.

Following the outline prescribed by the writer, a graduate student working under Professor Grinter was enabled to obtain a satisfactory analysis of a particular wind-stress problem to which he was assigned. This fact is gratifying evidence of the inherent accuracy of the method. "However," Professor

NOTE.—The paper by John E. Goldberg, Jun. Am. Soc. C. E., was published in May, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1933, by Messrs. G. A. Maney and L. E. Grinter; and January, 1934, by Messrs. S. S. Gorman, and A. Floris.

[16] Chicago, Ill.

[16a] Received by the Secretary February 20, 1934.

Grinter states, "finding nothing physical about the process * * *, he neither understood nor retained a knowledge of the procedure." Such an attitude is completely illogical and simply evidences the fact that no sincere effort was made to understand the method, because, of all the phenomena associated with the carrying of load by a single member, or by an integral structure, deformation and deflection alone have a physical and visual significance. Indisputably, slope-deflection methods possess an inherent advantage over other methods on this score. The tangibility is further increased by the method of working (as the writer suggested) directly on a line drawing of the frame under consideration.

Professor Maney points out the basic nature of the theory, and its simple and fundamental relation to convergence. Surely, neither slope-deflection theory nor convergence can be thought of as complicated. By its very nature, the proposed method possesses a physical tangibility that is indisputable.

Mr. Gorman places his finger on the crux of the matter. He writes, "the simplicity, rapidity, and accuracy which may be obtained in analyzing a frame is indeed surprising when one becomes adept with these operations."

Professor Grinter feels that the derivation of the formulas for the initial values of deflection is too long. There is a much shorter method of attack, but the writer presented the longer, more formal, derivation chiefly because it suggests the method by which the formulas for the initial value of the first-story joint rotation is obtained from fundamental considerations. However, it may be well, at this point, to outline the shorter derivation.

Assume, as in the paper, that all joints of the *n*th story of the bent under consideration, as well as of the *m*th and *o*th stories, have the same angular rotation, θ'_n. Then, by Equation (3), the moment at each end of each beam or girder of the *n*th story is, $-K_{Gn}\ (3\,\theta'_n)$, and the sum of the end moments for all the beams or girders of the *n*th story of the bent will be, $-\Sigma K_{Gn}\ (6\,\theta'_n)$. The sum of the column end moments of the *n*th story is M_n, and that of the *o*th story is M_o. Since at each joint of the *n*th story, $\Sigma_m = 0$, the same law must hold for all joints of the *n*th story taken together. Thus, $\frac{M_n}{2} + \frac{M_o}{2} - \Sigma K_{Gn}\ (6\,\theta'_n) = 0$; and, solving for θ'_n,

$$\theta'_n = \frac{M_n + M_o}{12\,\Sigma K_{Gn}} \qquad (16)$$

Direct substitution of θ' into the bent equation gives an expression for the initial value of the corresponding R'.

In presenting his conception of the composite bent, the writer expressly pointed out that it would find its most ready application in connection with the analysis of symmetrical and practically symmetrical buildings. However, Professor Grinter feels that it is a commendable contribution and he goes on to state that, "* * * it is natural that a deflection method should simplify the problem of the division of load between bents." Clearly, his statement points out an inherently advantageous characteristic of the method which the writer has proposed.

Mr, Gorman suggests a method for the analysis of symmetrical buildings which is so admirably straightforward that, no doubt, its value will be tested in the near future by application to actual problems. He objects to the statement that "it is usual to assume that the frame carries the entire [wind] load," and he cites an example in which the walls of a particular building carried about 90% of the lateral loading, leaving only a small amount for the frame. Obviously, such a structure as this should not be analyzed simply as a frame. The walls themselves should be designed and analyzed as the primary load-carrying units, and the frame should be considered more or less as a secondary effect, allocating to it a proportion of the total load based on some rational relationship between the action of the frame and the action of the walls; that is, deflections, etc.

Mr. Gorman suggests the temporary substitution of an equivalent single-span symmetrical bent for the actual bent. For the usual type of problem, requiring only a few cycles of approximations, exactly the same purpose is served simply by using the formulas of the paper for the initial values of joint rotation and story deflection, θ' and R'.

Mr. Floris refers to the approximate equations that the writer uses in the analysis. Possibly, the writer has misunderstood Mr. Floris' reference, but it is well to emphasize the fact that only the equations for θ' and R' are approximate formulas; all other equations used in the analysis are the correct slope-deflection equations.

Personal preference, no doubt, will be an important element in the choice of a method of wind-stress analysis. However, the attributes of the various methods should not be overlooked completely. Accuracy, is essential, of course. Workability has been added to correct basic theory to so great an extent that the use of methods of doubtful accuracy and undependable results no longer can be justified. Furthermore, a method that offers speed as well as the certainty of accuracy is to be preferred, naturally; and, again, wide scope and broad applicability should be important considerations. For example, a method that offers a rational means of analyzing complete buildings is preferable to a method of limited application.

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DISCUSSIONS

STABILITY OF STRAIGHT CONCRETE GRAVITY DAMS

Discussion

By Messrs. F. Knapp, and S. H. Woodard

F. Knapp,[45] Esq. (by letter).[45a]—On the basis of tests cited by the author it can be taken for granted that the internal friction theory as originated by Coulomb is applicable to dam design, at least in the range of sheer stresses usually found. The numerical values for the plane between the dam and the rock foundation remain to be determined. The experiments by S. H. Woodard, M. Am. Soc. C. E., mentioned by Mr. Henny, are not representative of this case. Pure shear tests are practically impossible because such tests always introduce bending moments and hence tension stresses. Since the shear strength under conditions of no load and the "factor of shearing strength increase" probably vary within wide limits for different qualities of concrete and rock, it seems best, under such circumstances, to arrange special tests for each single and important dam project in order to ascertain these factors. The principle of indirect load transmission is supposed to give practical results.

Fashioning a rather short test prism of the foundation material and pouring a similar prism of concrete with it monolithically (see Fig. 14) a test could be made that would duplicate exactly the conditions prevailing in the actual foundation. The samples would be tested in a vertical position, with horizontal forces, so that the bending moment in the plane between the two samples is zero and the shearing stress is maximum. Furthermore, the bond area between the two prisms could be subjected to water pressure simulating an uplift force. With such a testing arrangement the influence on the shear-

Note.—The paper by D. C. Henny, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by H. de B. Parsons, M. Am. Soc. C. E., December, 1933, by Messrs. A. A. Eremin and Calvin V. Davis; January, 1934, by Messrs. William P. Creager, F. W. Hanna, Lars R. Jorgensen, and I. M. Nelidov; and February, 1934, by Messrs. Paul Baumann, Thaddeus Merriman, Ivan E. Houk, A. V. Karpov, L. F. Harza, and Edward Godfrey.

[45] Asst. Hydr. Engr., The São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[45a] Received by the Secretary December 13, 1933.

ing strength between the dam and the foundation of all contributing factors (such as quality of rock and concrete, uplift and water-tightness, age of concrete, roughening of the bond area, etc.), could be determined

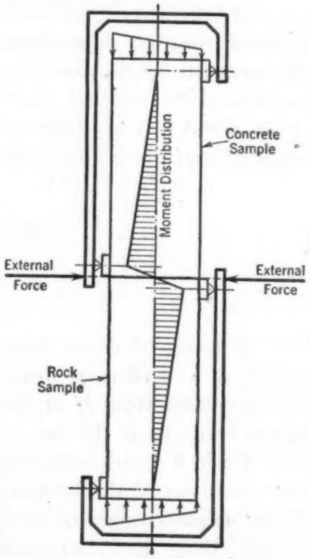

FIG. 14.—TESTING APPARATUS FOR DETERMINING SHEAR STRENGTH BETWEEN ROCK AND CONCRETE.

Concrete is a brittle material and, as such, no stress release is possible at overstressed places as would occur in the case of ductile materials. Consequently, the greatest shearing stress that develops in the dam should be taken for the determination of a shear safety factor. Equations (2), (6), and (13) do not comply with this requirement, but are based on the use of the total shearing force assumed to be distributed equally over the area. It would seem more logical to define this safety factor as the ratio of shear stress at failure to the maximum shear stress in the dam under the most severe condition of loading.

With the author's assumptions (namely, a triangular profile having a vertical water surface, horizontal base, and water at the apex level), the maximum normal and shearing stresses occur at the toe of the base. Equation (8) is also valid for the case of uplift considered; that is, with pressure decreasing from a maximum at the heel to zero at the toe of the base. In the 62.5-lb units used by Mr. Henny, the maximum shearing stress has a value of $\frac{H}{n}$, being maximum at the toe and decreasing uniformly to zero at the heel. The shear safety factor, therefore, should be defined as:

$$Q = \frac{s_1 + K\,q}{H}\,n = \frac{s_1\,n}{H} + \frac{K}{n} \quad\text{................}(26)$$

and, similarly, Equation (13) shows decreased factors of shear safety for increasing heights of the dam of constant relative width. It can also be shown that this factor is smallest at the toe of the base.

The author makes the implicit assumption that shear failure takes place simultaneously on the entire base of the dam. In the light of Equation (26), shear failure would begin at the heel and spread progressively toward the water face.

With values of 0.70 for K and 920 for s_1, Equation (26) gives, for the case of a dam with a relative base width of 0.707 H and for heights varying from 200 to 700 ft, the following factors of safety: 4.24, 3.16, 2.61, 2.29, 2.07, and 1.92 instead of 7.49, 5.32, 4.24, 3.59, 3.11, and 2.85 as given in Table 8, Column (3), Items Nos. 10 to 15.

The author's remark that the maximum shear stress will always be less than twice the average and that this range is fully covered by such assumptions as were made in his paper, is somewhat vague. Obviously, the derivation

of Equation (26) involves no assumptions regarding the distribution of stresses and strength, and for this reason the writer decidedly prefers such a treatment.

Mr. Henny states that the direct effect of drainage on uplift pressure is difficult to determine; that drains are likely to close and become clogged; and, that for these reasons conservatism in design dictates that no dependence should be placed on the perpetual reduction of uplift pressure by drains. The design of the Italian dam, Suviana,[46] is a good example of the caution taken to avoid clogging of drains. This dam has drain holes about 1 ft in diameter, spaced 20 ft apart. O. Hoffmann has shown the effect of drain holes upon the uplift and gives the following formula,[47]

$$\frac{H_1}{H} = \frac{1}{1 + \dfrac{\pi\, Y}{\dfrac{D \ln D}{\pi\, R}}} \qquad \text{(27)}$$

in which, H_1 = uplift pressure at the drain; Y = distance of drain from water face; $2D$ = center distance of the drains; and, R = radius of drain pipe or hole. According to Mr. Hoffmann the uplift decreases from H at the water face to H_1 at the drain and, from there, uniformly to zero at the toe.

The relative area subject to uplift as proposed in Table 6 varies with the heel loading, but does not take into account different qualities of the foundation. The German specifications[48] prescribe the following percentages of area subject to uplift: Good natural conditions of foundation, 20%; average natural conditions, 30%; and, less favorable natural conditions, 40 per cent.

It seems to the writer that for estimating the uplift in each particular case the judgment of an experienced engineer and geologist is still the safest and the natural way. In a remarkable paper, M. Lugeon[49] treats of the interrelation of dam design and geology and shows convincingly by means of several failures of dam projects that the co-operation of the engineer and the geologist in establishing the basis of the design is indispensable.

S. H. Woodard,[50] M. Am. Soc. C. E. (by letter)[50a].—For some time the writer has been impressed by the accumulation of evidence that, whatever the cause seemed to be—compression, or shear, or tension—all failures of concrete, mortar, and other like substances are due to tension. The usual failure of a concrete specimen subject to compression is nicely illustrated by Fig. 3 of the paper, which shows the typical conical break. The writer has been led to suspect that the conical breaks may be attributed to the restraining or hooping effect of the friction between the two heads of the machine and the two ends of the cylinder. To test this a large number of cylinders and prisms at the laboratories of the Alabama Power Company were broken in a compression machine having both ends of the cylinders or prisms well lubricated. This

[46] "Impianti idroelettrica nella regione appeninica Tosca-Emiliana, alta valle del Reno e Limatra," *Energia Elettrica*, 1925.

[47] "Permeazioni d'acqua e lor effeti nei muri de ritenuta," O. Hoffmann, Milano, 1928.

[48] "Anleitung für den Entwurf, Bau, and Betrieb von Talsperren," Berlin, 1930.

[49] "Barrages et Géologie," M. Lugeon, Lausanne, 1933.

[50] Cons. Engr., New York, N. Y.

[50a] Received by the Secretary February 7, 1934.

was done in various ways, one of the most satisfactory being the use of blotting paper impregnated with heavy grease inserted between the ends of the specimen and the heads of the machine. The specimens cracked vertically from end to end, forming small columns. The results of one set of such tests are given in Table 10.

TABLE 10.—TESTS OF TENSILE AND COMPRESSION SAMPLES OF PLAIN CONCRETE

Batch No.	Average tensile strength, in pounds per square inch	SPECIMEN		GREASED ENDS				PLAIN ENDS		
		Type	Dimensions, in inches	Compression strength, C, in pounds per square inch*	Type of break†	$\frac{t}{c} = n$: Each item	$\frac{t}{c} = n$: Mean	Compression strength, K, in pounds per square inch	Type of break†	$\frac{K}{C}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
				(*a*) 1 : 3 MORTAR SPECIMENS						
2	443	Cylinders	2 by 8	2 480	Col	0.178	0.148	3 430	Con	1.40
2	443	Prisms	4 by 4 by 16	3 750	Col	0.118		3 930	Pyr	1.05
4	419	Cylinders	2 by 6	2 944	Col	0.143	0.145	4 237	Con	1.44
4	419	Prisms	4 by 4 by 8	2 867	Col	0.147		4 105	Pyr	1.40
6	436	Cylinders	2 by 3	3 185	Col	0.138	0.159	3 892	Con	1.20
6	436	Prisms	4 by 4 by 6	2 420	Col	0.180		4 235	Pyr	1.75
8	459	Cylinders	2 by 2	3 603	Col	0.126	0.142	4 014	Con	1.11
8	459	Cubes	4 by 4 by 4	2 880	Col	0.159		4 045	Pyr	1.55
				(*b*) NEAT CEMENT SPECIMENS						
1	777	Cylinders	2 by 8	9 087	Col	0.086	0.098	12 013	Con	1.20
1	777	Prisms	4 by 4 by 16	6 170	Col	0.110		6 935	Pyr	1.12
3	678	Cylinders	2 by 4	6 300	Col	0.123	0.117	11 187	Con	1.78
3	678	Prisms	4 by 4 by 8	6 020	Col	0.112		7 745	Pyr	1.28
5	634	Cylinders	2 by 3	6 483	Col	0.098	0.097	11 667	Con	1.80
5	634	Prisms	4 by 4 by 6	6 584	Col	0.096		10 220	Pyr	1.56
7	680	Cylinders	2 by 2	6 122	Col	0.111	0.125	11 160	Con	1.83
7	680	Cubes	4 by 4 by 4	4 882	Col	0.138		9 662	Pyr	1.95

* All specimens cracked vertically, forming small columns. † "Col" denotes a columnar break; "Con" denotes a conical break; and "Pyr" denotes a pyramidal break.

A set of 180 cylinders and prisms was made for this test, which were from 2 to 6 in. in diameter, with lengths varying from one to four times their diameter. One-half the samples (that is, 90) were made of neat cement mortar (Table 10 (*b*)), and 90 were made of 1 part cement and 3 parts of standard Ottawa sand (Table 10 (*a*)). From each batch of mortar a set of nine tensile briquettes was moulded at the same time that the compression specimens were made. As there were ten batches there were 90 control tensile briquettes and 180 compression samples.

All specimens were broken at the end of 28 days. One-half of all the compression specimens were broken in the ordinary way without greasing the ends, and the remaining 90 (45 neat and 45 mortar specimens) were broken with greased tops and bottoms. All the latter resulted in columnar breaks and all the former in the usual cone or pyramid at the two ends. These results suggest a tentative law that, normal to the direction of every compressive stress, there is a tensile stress, and that the ratio between the magnitude of the tensile and compression stresses will be a constant factor char-

acteristic of the material. Thus, if p equals a compressive stress, in pounds per square inch, and f equals resulting tensile stress, in pounds per square inch,

$$f = -np \quad \text{..........................(28)}$$

From Table 10(a), it appears that the value of n for those particular samples of 1:3 mortar was between 0.14 and 0.159, while the value for the stronger neat cement mortar (Table 10(b)) was between 0.097 and 0.125.

All materials subject to shortening of one axis by simple compression are known to swell or to elongate in directions normal to that axis the amount represented by Poisson's ratio, and it may be expected that elongation would be accompanied by tension. In Equation (28), n is a ratio of stresses and, therefore, is not Poisson's, which is a ratio of strains; but if the moduli of elasticity are constant the values of the two might be the same. The values of n in Table 10 are near enough to the known values of Poisson's ratio for mortar and concrete to be at least suggestive. As is usual with tensile breaks, individual briquettes broke at tensile values 10% greater and less than the averages shown in Column (2), Table 10, and it is interesting to note that if values of n equal 0.148 for 1:3 mortar and 0.109 for neat cement mortar had been used in Equation (28) to determine the tensile strength from the results of the compression tests with greased ends, the resulting determination of tensile strengths would have been as close to the average of the tensile breaks as were the individual tensile breaks themselves.

The writer is rather diffident about reporting these experiments because the results are incomplete and somewhat erratic. The series was designed as a pilot set of experiments to furnish a guide for a much more elaborate series which had to be abandoned. Further research is wanted to settle, definitely, the question of whether the greased bearing method of testing compression samples does not give a truer determination of the strength of the material under compression than the method now commonly used. Further research would also be required to establish, thoroughly, any such law as is tentatively suggested herein.

A mental picture of the physical operations of such a law may be formed by considering the materials under compression to be made up of particles that are connected by an elastic tensile strength (which may be conceived to be cementation, attraction, adherence, or any elastic holding property) and separated or prevented from nearer approach to each other by a relatively great resistance which may or may not have some elastic properties. These particles may be conceived as anything from Bohr atoms to grains of sand. If they are packed in an amorphous state into any given space, the transfer of force from one side of that space to the other would seem to be as if by tetrahedral frames, the junction points being at the centers of the particles and the members having definite tensile properties and relatively extremely great compressive strength. In such a structure compression in any direction means tension normal to that direction. This may not be a true or complete picture of what happens, but it does illustrate a possibility that would explain the relationship of compressive and tensile stresses.

Referring now to the set of experiments, summarized in Table 3 and Fig. 2, a set of 6 by 12-in. prisms was cast of neat cement mortar, except for a $\frac{3}{4}$-in. band of mortar made of 1 part cement to 3 parts of standard Ottawa sand. This was done by setting the wooden mould in an inclined position so that the desired shear band would be horizontal. The mould was filled with neat cement mortar up to the under side of the desired shear band and immediately the $\frac{3}{4}$-in. layer of 1:3 mortar was placed and the remainder of the volume filled with neat cement mortar. The wooden moulds were especially designed and built in three parts, separated at the planes of the two sides of the $\frac{3}{4}$-in. shear band to permit this to be done without distortion. When the moulds were removed there was no indication that the shear band had been distorted. From each batch of 1:3 mortar which was used to make the shear bands a control set of six tensile briquettes was made, except that for Specimens Nos. 18 and 20 only two each were made. No control samples were made for the neat cement mortar, which is to be regretted.

The fundamental data of Table 11 are the same as in Table 3, with the addition of the breaking strength of the control tensile briquettes. In Column (5), $p = \frac{P}{A}$, of course, is found by dividing Column (3) by Column (4). Items Nos. 15 and 18 (Column (5)) were computed by the formula,

$$p = \frac{P}{2\,a \sin \alpha \cos \beta} \quad \text{.....................(29)}$$

in which, a = the area of the shear plane and β = the angle between the direction of the resultant and that of the load.

TABLE 11.—TESTS OF STRESS ON DIAGONAL SHEAR PLANES

Specimen No. (1)	cot α (2)	Total compression, P, in pounds (3)	Area, A, in square inches (4)	Values of P, in pounds per square inch (5)	Values of f_1, in pounds per square inch (6)	Tensile strength, t (1:3 mortar), in pounds per square inch (7)
1	0.40	145 650	36	4 046	—218	—420
2	0.40	156 270	36	4 341	—240	—338
3	0.80	138 850	36	3 857	—350	—420
4	0.80	141 070	36	3 911	—354	—338
5	0.80	141 170	36	3 921	—355	—312
6	1.20	115 170	36	3 199	—356	—420
7	1.20	95 210	36	2 645	—297	—338
8	1.20	98 320	36	2 731	—305	—312
9	1.75	86 340	36	2 395	—302	—420
10	1.75	99 520	36	2 765	—348	—338
11	1.75	90 300	36	2 508	—316	—312
12	2.33	99 700	36	2 770	—368	—420
13	2.33	91 350	36	2 537	—338	—338
14	2.33	81 880	36	2 275	—303	—312
15	7.95	50 820	..	2 812	—401	—420
16	7.95	50 600	..	2 800	—397	—338
17	7.95	49 090	..	2 720	—386	—312
18	5.15	57 610	..	2 120	—301	—221
19	5.15	66 670	..	2 245	—348	—305
20	5.15	78 480	..	2 880	—407	—277

When making the tests, as soon as the beam of the testing machine dropped, indicating that a specimen had failed, a tracing was made of the cracks showing on the sides of the specimen (see Fig. 15). A line of thrust

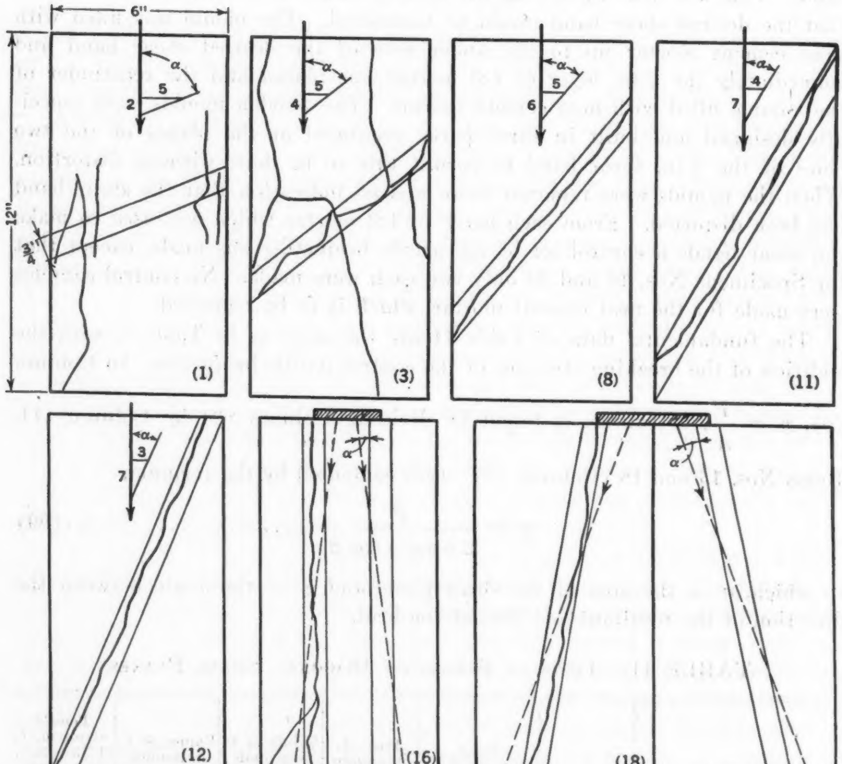

FIG. 15.

is indicated on Specimens Nos. 16 and 18. On the other specimens (Nos. 1 to 14, inclusive), the line of thrust, of course, is normal to the bed of the testing machine. The exact position of the line of thrust in Specimens Nos. 15 to 20, inclusive, can only be approximated. A change of position would change, somewhat, the values of f_1 given in Table 11, Column (6), for Specimens Nos. 15 to 20.

Column (6), Table 11, shows the computed tensile stress across the shear planes from the formula,

$$f_1 = -np\cos\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

in which, α is the angle between the shear plane and the direction of the compressive forces; p, the compressive stress; and n, as before, a constant which is a characteristic of the material and is the ratio between the tensile stress

(set up by the compressive stress normal to the direction of compression) and the compressive stress. For computing Column (6), the value of n is taken to be 0.145, which is a fair average of the values of n for 1:3 mortar shown in Table 10(a). Of course, as this is a factor assumed to be characteristic of the material and as six different batches of 1:3 mortar were used, it is probable that there were six values of n differing slightly among themselves.

The values of f_1 for all specimens that have clear breaks in the shear band are close enough to the values of t to make it seem probable that the formula is at least approximate. If this is so, f_1 is the component of the stress, f, normal to the direction of compression when that stress is resolved normal to the shear plane.

If there is a tensile stress across the shear plane which is a function of the principal compressive stress then it is a function of the components of the principal stress. While this relation is not a simple one (as the author points out), the formulas,

$$f_1 = 0.35q - 0.55s \qquad (31)$$

or,

$$f_1 = 035p \sin^2 \alpha - 0.55p \sin \alpha \cos \alpha \qquad (32)$$

will give values of $- f_1$ which are consistent with the results of the tests of Table 11. In Equations (31) and (32), $q =$ the unit compression on the shear plane, and s is the unit shear.

While the values of f_1 computed from Equations (30) and (31) are consistent with the breaks, they are not consistent with each other because, when α is greater than about 68°, Equation (30) gives negative values and Equation (31) gives positive values. As the specimens having shear planes at that angle failed in the neat cement mortar (Specimens Nos. 1, 2, 3, 6, and 9, in Column (7) Table 10), what would have happened in the shear band is not known.

If the force, P, is resolved into two components, Q, normal to the shear planes (producing the compressive stress, q, on the shear plane), and V, in a direction of the shear plane (producing a compressive stress, v, on a plane normal to the shear plane), the equation,

$$f_1 = - \frac{2\,nq}{3} - n\,v \qquad (33)$$

or

$$f_1 = - \frac{2\,np \sin^2 \alpha}{3} - np \cos^2 \alpha \qquad (34)$$

will also give values of f_1 that will be consistent with the results of the breaks of the specimens in Table 11.

However, the experiments are too crude and limited in number to establish any formula and there is uncertainty as to the effect of the probable higher modulus of elasticity for the neat cement portion of the prisms. It is hoped

that they may incite some one to frame other experiments which will give more conclusive results.

The experiments on the force of water in the pores of mortar referred to in Table 7 are probably not conclusive. They should be repeated, with a determination of the time element. However, in gravity dams, the danger is principally from water pressure in horizontal construction joints in which the cross-section of pores may be many times the cross-section of pores in the adjacent concrete.

With reference to construction joints between the layers of successive concrete placing in gravity dams, the writer has adopted the practice of sloping them downward from the down-stream face to the up-stream face of the dam; thus, they are not in the critical section, which is horizontal. It is found that concrete can be placed on a slope of 1 on 10 as conveniently as horizontal.

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DISCUSSIONS

ESTIMATING THE ECONOMIC VALUE OF PROPOSED HIGHWAY EXPENDITURES

Discussion

BY MESSRS. H. E. PHELPS, AND C. C. WILEY

H. E. PHELPS,[24] M. AM. SOC. C. E. (by letter)[24a].—Old data relating to the cost of vehicle operation cannot be used unchanged in economic comparisons of various types of surfaces, when one of the surfaces is either bituminous treated gravel, mixed-in-place crushed stone surface, or of the plant-mixed type. These surfaces are "intermediate" as regards cost, but all available evidence places them very close to pavements as regards vehicle-operating costs.

This evidence falls into two classes, dealing with gasoline consumption and tire wear. As to gasoline the latest evidence is given by R. J. Paustian, Jun. Am. Soc. C. E.[25] From Fig. 2 it may be concluded: (*a*) That the roughness of concrete surfaces has no appreciable effect on gasoline consumption; (*b*) that bituminous treated gravel is better than wet concrete for speeds less than 38 miles per hr; (*c*) that at a road speed of 40 miles per hr bituminous treated gravel has a fuel consumption of 1.06 (approximately) as compared with concrete (wet or dry), reducing to an inappreciable difference at speeds between 25 and 30 miles per hr (as compared with 1.20 in Table 1); and (*d*) that bituminous treated gravel is much more closely a high type than an intermediate type as regards fuel consumption.

There is little real evidence that tire costs are any greater on bituminous treated surfaces than on concrete. The cost of tires as an item of vehicle-operating costs has no very direct relationship to the item of tire wear, since accidents and age rather than wear frequently necessitate tire replacements. Tire wear tests at the State College of Washington do show greatly increased wear on untreated gravel surfaces consisting of sharp-edged basaltic rocks, but they also indicate very strongly that tire wear on bituminous, surface-treated gravel roads is at least no greater than on concrete.

NOTE.—The paper by Thomas R. Agg, M. Am. Soc. C. E., was presented at the meeting of the Highway Division, Atlantic City, N. J., October 10, 1932, and published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. W. W. Crosby, Roger L. Morrison, and Samuel B. Folk; December, 1933, by Messrs. W. S. Downs and George E. Martin; and January, 1934, by J. T. L. McNew, M. Am. Soc. C. E.

[24] Prof., Highway Eng., State Coll. of Washington, Pullman, Wash.

[24a] Received by the Secretary January 9, 1934.

[25] *Civil Engineering*, July, 1933, p. 377, Fig. 2.

A. A. Anderson, Assoc. M. Am. Soc. C. E., and Mr. H. B. Wright have reported[26] 81% greater tire wear on non skid asphaltic concrete than on concrete, but W. C. McNown, M. Am. Soc. C. E., reports[27] that tire wear is nearly the

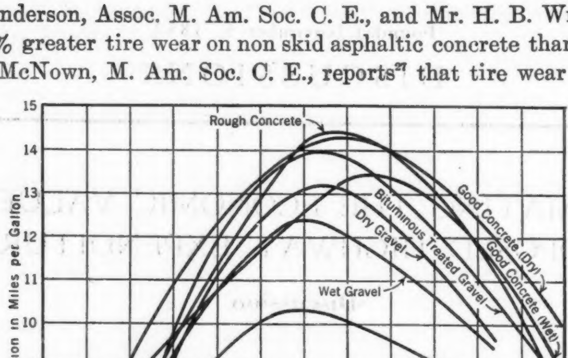

FIG. 2.—EFFECT OF ROAD SURFACE ON GASOLINE CONSUMPTION

same on all firm-surface roads. This evidence indicates that tire wear is much more closely related to the texture of the surface than to either its roughness or its kind, and that file-like surfaces such as exist on some of the open non-skid bituminous mixtures or on Portland cement finish by brooming or wiping will produce greater tire wear than the more smoothly traveled surfaces of earlier concrete pavements. There is no evidence that modern oiled roads or asphaltic concrete surfaces cause either greater or less tire wear than modern concrete pavements, and until this evidence is forthcoming it is futile to attempt to draw valid conclusions as to the relative economic merits of different types of pavements.

C. C. WILEY,[28] M. AM. SOC. C. E. (by letter)[28a].—The economics of highway expenditures is a problem that is both elusive and confusing. The fundamental concepts are so entangled with governmental processes, sociological requirements, and financial operations that it is almost impossible to trace them, while many of the factors necessary to a solution are as yet unknown and perhaps even impossible of determination. Economic writers seem to have severed this Gordian knot by the simple expedient of waving aside all but the financial aspects, and then to have treated these aspects by conventional methods. The conclusions reached may therefore be far from reliable.

The present-day discussion of highway economics seems to have started about twenty-five years ago when the U. S. Bureau of Public Roads, in its

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[26] Rept., 11th Annual Meeting, Highway Research Board.

[27] Rept., 6th Annual Meeting, Highway Research Board.

[28] Assoc. Prof., Highway Eng., Univ. of Illinois, Urbana, Ill.

[28a] Received by the Secretary January 26, 1934.

efforts to stimulate interest in better highways, pointed out that the capitalized value of the decrease in hauling costs due to an improved surface would generally be more than the cost of the improvement and, therefore, the many benefits of good roads, both social and economic, were, in effect, to be had without additional expense.

From that simple beginning there appears to have evolved a current theory of highway economics based on the two major assumptions: (1) That road improvements must be justified by reduced transportation costs; and (2) that such costs are to be computed as if the entire expenditures for road construction were capital investments in a commercial enterprise for profit.

The first assumption is in error because it makes use of only one of several economic benefits of good roads and entirely ignores the social advantages. It is true that some writers admit that there are other than financial considerations, but the general inference, whether intentional or not, is that they are relatively unimportant. Other writers, however, state positively that the improvements must be justified by reduced costs of transportation.[29]

This assumption is approximately correct, however, when it is preceded by, or based on, the additional assumption that each type of road under consideration can render the required basic service. Costs of transportation then become the distinguishing feature between the various types. If the various kinds of roads can not render identical service then all advantages must be considered and the social benefits may become the controlling factor, with hauling costs the least important in the selection of the type to use. The point that seems to have been lost is that if reduced transportation costs are sufficient to finance the improvement, all the other benefits are obtained for nothing. The mere fact that the savings in hauling costs will not pay for the improvement does not necessarily make the improvement a luxury.[29] The proper assumption is that, other things being equal, the improvement should be justified by reduced transportation costs and then extreme care must be taken to see that the "other things" actually are equal.

The second assumption seems to be the outgrowth of borrowing the ordinary formula (the author's Equation (2)) for theoretical investments without regard to the actual conditions that surround the financing of road improvements by governmental bodies. The net result is that this assumption is fundamentally wrong.

Highway expenditures are in no sense capital investments. They are the direct cash outlay by the people for the purchase of a service perhaps as fundamental to present civilization as government itself. If, however, some improvement in the service results in a saving to those receiving it then this saving, and this saving only, may be considered as available for a limited time to pay for the improvement. It must be remembered, however, that the cost of a road improvement (including interest on bonds, if any), is actually paid out by the people in the form of taxes, while the savings accrue later by the reduced costs in the purchase of other items incidental to the use of the road.

[29] "Highway Construction, Administration, and Finance," by E. W. James, M. Am. Soc. C. E., Highway Education Board, Washington, D. C.

Neither the governmental units that build the roads nor the taxpayers who pay for them can be considered as investors in the usual commercial sense. Government is essentially a "corporation not for profit"; and hence it is not an investing body. To accept the author's statement that "had that sum remained in the hands of the individuals who comprise the public, each might have invested it in a dividend or interest paying security," is to establish the principle that capital interest can be charged on tax payments. This is in direct violation of all business practices whereby taxes are invariably written off as items of expense. The entire theory of computing highway costs as capital investments fails on this one point alone.

As further evidence, it would be absolutely essential, in order to make the theory tenable, that each and every taxpayer would have equal and ample facilities for the safe, easy, and adequate investment of his funds. No such possibility exists and probably never could exist. What a "thrifty" few may do under existing conditions can not be assumed to be what all could do under other conditions. Government or road bonds could not be depended upon since there are not enough of them. When one person buys a bond some one else must sell; hence, there would have to be enough new bonds each year to absorb the road funds for that year. Consequently, recourse would have to be made to other kinds of investments with all their variable characteristics of safety, convenience, and adequacy of return. The only sane way to invest tax funds is to pay the taxes, and no other investment yields as great a return, using the term, "investment," in its broad sense.

The commercial method of computing highway costs has the further fault that it is based on the use of annuities. Past experience with accumulations of public funds have been so consistently disastrous that their use is rapidly disappearing. Sinking-fund transactions by public bodies are specifically barred by law in some States. It seems to be a growing principle of taxation that taxes shall be levied only for current expenditures (including debts due) and not for accumulation. It is scarcely logical to base computations on a type of transactions that legally can not be made.

Incidentally, the item for depreciation in the cost formula (Equation (2)), has been confusing to many people. The attitude taken is that the road has been paid for, and to charge depreciation is making them pay for it twice. This is not correct, of course, because the depreciation item is merely the device for including the first cost; but instead of including the actual cost a fictitious amount in the form of an annuity is introduced.

The proper consideration of bond interest has been confusing only because the true function of the bond issue has been overlooked. Road bonds are used when it is desired to complete a definite system in a short time so that the generation paying the bill can have the use of the entire system while paying for it. Thus, a county that needs 200 miles of improved roads for adequate service to the people, but can build only 10 miles per year from current funds, would require 20 years to build the system and at no time during this period would the entire mileage be available for use. If, however, bonds were issued and the roads were built in 2 years, the entire system

would be available for 18 of the 20 years. To do this would require the payment of interest which, therefore, would become merely a premium paid for the privilege of having the system now instead of twenty years from now. Bond interest, therefore, is simply an added item of first cost and should appear as such in computing either the total or the annual costs.[29]

The proper calculation of highway costs should be based only on the actual sums involved. The equation for average annual cost of a road improvement, therefore, becomes,

$$C = \frac{I - S + i + M + A}{N} \quad \text{.................} (4)$$

in which, C is the average annual cost at the age of N years; N, the age, in years, over which the cost is distributed; I, the initial cost of the project, including financing, construction, engineering, and administration; S, the salvage value at the age of N years; i, the total interest paid in the period, N years; M, the total maintenance cost in the period, N years, including special or periodic repairs; and, A, the total administrative and other costs during the period, N years.

The investment idea persists in computing vehicle costs in the form of an item for annual interest. This practice is probably correct when the vehicles are essential items in the capital investment for the equipment of a commercial enterprise, such as a road contractor or trucking company. It is questionable, however, when the vehicles are merely substituted for some other form of transportation expense (as would be the case when a business concern supplies its salesmen with cars to replace railroad travel, and which is probably the correct status of the majority of private cars). If the car was purchased on deferred payments, the financing charges would have to be included. This item of interest, however, is not large, and since the costs are based on broad averages from which there is a wide range of variation, it probably does not seriously affect the accuracy of the vehicle costs per mile.

The two examples of highway costs given by Dean Agg are worth a little study. In the case of the gravel road, the total cost includes all items, and this is also probably true of the maintenance charge since no contrary statement is made. The 3-year periodic repairs would probably be made by dividing the 26-mile line into three sections, and repairing one section each year from current funds at a cost of \$440 per mile, instead of accumulating a sufficient fund every 3 years by an annuity of \$422 per mile. Possibly, the money would be borrowed, which would add an interest item to the cost. Depreciation appears to be computed only on the gravel surface; hence, the other items included in the total cost must be presumed not to depreciate.

Turning, now, to the concrete road it is found that, again, total costs and maintenance costs cover all items. Periodic repairs, however, are considered as occurring only once in twenty years and then in the form of a new surface on the old concrete as a base. The sum allowed for this purpose obviously will not replace the surface with concrete; hence the pavement has

now changed type. It is no longer a concrete pavement; consequently, the original "investment" has ended and a new computation based on the new type must be started, just as when the change was made from gravel to concrete.

Depreciation has been computed on a sum found by subtracting the salvage value of the concrete surface from the total cost of the road. Therefore, it includes 100% depreciation on earthwork, right of way, etc., which obviously is not correct. In both cases the life of these items should be estimated, and the cost prorated over the term considered.

Using Equation (4) and including, for simplicity, only the items relating to the surface, the cost of the gravel road is,

$$C = \frac{84\,700 + 20\,775 - 0 + 0 + 20 \times 21\,500 + 6 \times 34\,300}{20} = \$37\,064$$

or $1 425 per mile. In the case of the concrete road,

$$C = \frac{817\,260 - 463\,500 + 0 + 20 \times 24\,380}{20} = \$42\,068$$

or $1 618 per mile.

These values combined with the given vehicle costs yield 6.40 and 5.45 cents, respectively, for the comparative transportation costs on the gravel and concrete. This gives a difference of 0.95 cent per vehicle-mile in favor of the concrete. Considering this difference to apply to the entire 700 000 vehicles per year for the full 26 miles there results an annual saving of $172 900. This is the annual sum that may be considered investable in the improvement because it actually remains in the hands of the people.

In computing such investments it is customary to consider the annual saving as an annuity and to determine its present value for a given term at a given interest rate. Since annuity bonds are little used, and since in many States serial bonds are required by law, it would seem more logical to determine the amount of serial bonds on which the annual saving would equal the maximum annual payment of principle and interest. This amount is given by the equation,

$$P = \frac{S}{\frac{1}{N} + r} \quad \text{.....................(5)}$$

With $r = 0.04$ and $N = 20$, the amount of bonds which the annual saving of $172 900 can handle, is $1 921 111, instead of $2 349 767 as given by the annuity method. This is the sum which, theoretically, can be borrowed to change the road from gravel to concrete. Since the cost of 26 miles of concrete was only $817 250, it is evident that the improvement is justified. Since the total cost was only $1 216 877, the saving will pay for the entire road which is more than it is required to do. By using $P = \$817\,250$ and solving for N it is found that the saving would pay for the change from gravel to concrete in 7 years with 4% serial bonds.

The examples of decreased distance and grade reduction both illustrate the investment of the savings resulting from improvements. The first is valuable because of the figures presented for the cost of short changes of distance. The second is an excellent brief outline of the plan of attack of the grade-reduction problem. Great care is needed in any particular case in analyzing the conditions and in evaluating the factors to be used.

The only other point which needs consideration is the item of time. The value of the time factor has been greatly exaggerated in all discussions of road and street economics. Small periods of time are valueless because they can not be used. Thus, the saving of 1 min or 2 min to each vehicle on a given road is no economic gain, because this small increment of time can not be put to use. There is a vast difference in the economic value of the delay of 1 min to each of 60 cars and the delay of 60 min to 1 car, although the "car-minutes" are the same. The customary practice of multiplying these small fractions of time by the large number of people or vehicles to which they accrue individually, and assigning a value to the product, is erroneous. It is only when the fractions of time to some individual become of usable size that they become of value, and then only to that individual. Just what minimum period of time begins to have an economic value is unknown. It is probably different for every location and for every vehicle that passes that location.

It must be quite evident to even a casual reader that the economics of highway improvements are not, as yet, established. Many factors must be evaluated, and the entire problem must be carefully analyzed. There is a fine field for further work in this line. Meanwhile, highway engineers should not take any theory or data too seriously lest they deceive themselves and mislead the public.

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DISCUSSIONS

INTERCEPTING SEWERS AND STORM STAND-BY TANKS AT COLUMBUS, OHIO

Discussion

BY MESSRS. D. T. MITCHELL, ROBERT SPURR WESTON, AND W. W. HORNER

D. T. MITCHELL,[14] ASSOC. M. AM. SOC. C. E. (by letter)[14a].—Little space has been alloted in this paper to description of the actual construction. Having been associated with the authors on the construction of the intercepting sewers and regulator chambers, the writer feels that a discussion of this phase will be of interest.

For the majority of the intercepting sewers and regulator chambers the concrete was mixed for 3 min in 1-cu yd mixers of the paving type. For sewers, Types *L* and *M* (Fig. 5) and the Broad Street and Chestnut Street Regulator Chambers, however, truck mixers were used, while for Type *R* sewer (Fig. 8) and for a part of Types *A* and *P*, the concrete was mixed at a central mixing plant from which it was transported, in trucks, equipped with mechanical agitators, to the site of the work. Where truck mixers were used, the concrete was mixed for 5 min.

The mix was transported to the forms by buckets, buggies, or cars. On some of the work, a pipe held in a vertical or substantially vertical position, which was kept full at all times during the placing of concrete, was used. Such a pipe, however, was not permitted to discharge directly into the forms, but was used to transport the concrete from the mixer to a hopper or to the buggies or cars. Individual chutes longer than 8 ft, or a system of chutes the aggregate length of which was more than 8 ft, were not permitted.

The concrete for sewers, Type *D* and part of Types *A* and *C* (Fig. 4), was made in the proportions of 1: 5.5; that for Types *N* and *O*, in which high early strength to permit back-filling in 4 days was desired, in the proportions

NOTE.—The paper by John H. Gregory, R. H. Simpson, Orris Bonney, and Robert A. Allton, Members, Am. Soc. C. E., was published in October, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: February, 1934, by Messrs. C. B. Hoover and C. D. McGuire, and Julian Montgomery.

[14] Field Engr., Sewer Dept., Div. of Eng. and Constr., City of Columbus, Columbus, Ohio.

[14a] Received by the Secretary, January 15. 1934.

of 1 volume of cement to 4 volumes of aggregate; and that for the cradle and backing under and around the vitrified clay segmental block sewer of Type *S*, in the proportion of 1 volume of cement to 12 volumes of aggregate. For the other types of sewer and for the Broad Street and Chestnut Street Regulator Chambers, the concrete was made in the proportions of 1 volume of cement to 6 volumes of aggregate; and for the other regulator chambers, in the proportion of 1 volume of cement to 5.5 volumes of aggregate. On Types *N* and *O*, an average 4-day compressive strength of 2 700 lb per sq in. was obtained.

TABLE 8.—UNIT CONTRACT PRICES FOR THE PRINCIPAL ITEMS.

Type	Year	Excavation per cubic yard	Timber piles per foot of pile	Steel sheet-piling per square foot of wall	Reinforcing steel per pound	Concrete per cubic yard	Vitrified clay liner plates per square foot	Miscellaneous cast-iron and steel per pound
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(a) OLENTANGY-SCIOTO INTERCEPTING SEWER								
A	1932	$1.20	....	....	$0.03	$9.00	0.20	$0.05
	1930	1.20	....	....	0.035	10.85	0.183	0.12
B	1930	1.20	....	....	0.035	10.85	0.183**	0.12
C	1930	1.20	$0.60	$1.25	0.035	10.85	0.183**	0.12
	1930	0.45–0.50*	0.60‡	1.25	0.0305	10.00	0.18	0.08
D	1930	0.45–0.50*	2.10‡	....	0.0305	10.00	0.18	0.08
E	1930	1.05	....	....	0.035	10.50	0.188**	....
F	1930	1.05	....	....	0.035	10.50	0.188**	....
G	1928	1.75	....	....	0.0375	11.50	0.204**	....
H	1928	1.75	....	....	0.0375	11.50	0.204**	....
I	1928	1.75	....	....	0.0375	11.50	0.204**	....
J	1929	1.20–1.30*	0.51	0.037§	0.04	11.50	0.188**	0.15
K	1929	1.20–1.30*	0.51	0.037§	0.04	11.50	0.188**	0.15
L	1930	8.38	....	....	0.04	14.00	0.20	0.13
M	1930	8.38	....	....	0.04	14.00	0.20	0.13
N	1932	1.40	....	....	0.04	6.26	0.15	....
O	1932	1.40	....	....	0.04	6.26	0.15	....
(b) ALUM CREEK INTERCEPTING SEWER								
P	1929	$21.00†	....	....	$2.60†	$18.00‖	....	....
Q	1929	21.00†	....	....	....	18.00‖	....	....
R	1929	....	....	....	....	42.00¶	....	....
S	1930	1.00	....	....	....	6.75	$5.50††	....
(c) REGULATOR CHAMBERS								
Broad Street, Chestnut St.	1930	$5.62	....	....	$0.04	$21.25	$0.20	$0.13
Peters Run, Rich Street, Town Street, Henry Street	1931	2.40	$2.00	....	$0.038	$12.40	$0.20	$0.15
First Avenue	1932	1.20	....	....	0.04	11.00	0.15	0.04

* Wet excavation. † Price per linear foot of sewer. ‡ Concrete piles. § Per pound. ‖ Price per linear foot of sewer, including liner-plates. ¶ Total price for complete sewer per linear foot. ** Computed from prices per linear foot of sewer. †† Price per linear foot of sewer for segment block.

Throughout the work, special attention was directed toward the making of concrete of uniform quality. In 1931, data pertaining to the breaking of 1051 test cylinders,[45] representing 34 127 cu yd of concrete made in the proportions of 1 volume of cement to 6 volumes of aggregate, were tabulated. The average breaking strength at 28 days of the 1 051 cylinders was 3 371 lb per sq in., the average for cylinders testing less than that amount was 2 901 lb per sq in., and the average for cylinders testing more was 3 877. Of

the 1 051 cylinders, 52% tested less than the average strength and only 2% tested less than 2 000 lb per sq in. Comparable results were obtained on the remainder of the work.

Ordinarily, wall forms for the concrete in the rectangular sewers and regulator chambers were held in place 4 days, while those for the roof slabs in the rectangular sewers were held for 6 days, and those for the regulator chambers, 7 days. The invert and arch forms for Types *L* and *M* were held 3 days, and those for Types *P* and *Q*, 1 day. For Types *N* and *O*, the invert forms were held 15 hr and the arch forms, 2 days, while on Type *R* the invert form was held 20 hr and that for the arch, 2 days.

Vitrified clay liner plates in the arches of the circular sewers and in the walls and roofs of the rectangular sewers were attached to the forms and the concrete was placed against them. In the lower part of the circular sewers, and in the bottom of the rectangular sewers, the plates were laid in mortar after the concrete had been placed.

The writer feels that cost data in more detail than those given by the authors are of interest and, therefore, has included in Table 8, the unit prices for the principal items involved in the construction of the intercepting sewers and regulator chambers.

ROBERT SPURR WESTON,[15] M. AM. SOC. C. E. (by letter)[15a].—The sanitary engineering history of Columbus, Ohio, is a most interesting one, especially to the writer, whose first visit was made more than thirty years ago when the late Rudolph Hering, M. Am. Soc. C. E., was reviewing the earlier work of John W. Alvord, M. Am. Soc. C. E., and the late Julian Griggs, M. Am. Soc. C. E. In those days prevailing winds brought offensive odors to the heart of the city from the putrefying pools in the Scioto River just below, and the typhoid fever death rate was sometimes of epidemic degree.

The problems of Columbus are those of a large and growing city situated on small rivers where, as elsewhere, there is a strong trend away from excessive concentration of population in the municipal center, and also a tendency to extend, by annexation and other arrangements, the muncipal utilities to surrounding suburban areas. Columbus is typical in this respect, and the authorities have acted wisely in designing for populations in 1960 which, at this time, hardly seem realizable.

Interesting as are the intercepting sewers and the carefully designed control works, the stand-by tanks are of greatest interest because new in practice.

Those who study rivers and the condition of river water, are beginning to realize how easy it is for deposits to form near the mouths of sewers, especially combined sewers, even in considerable streams; how the sand and silt deposited from sewage entrains with it putrescible organic matter; how little is the cleansing effect of scour in some rivers; how much reliance must be placed upon the self-purification of a stream by digestion; and how foul a stream becomes when its digestive capacity is exceeded. With these facts in

[15] Cons. Engr. (Weston & Sampson), Boston, Mass.

[15a] Received by the Secretary January 30, 1934.

mind, even if no economies in construction were effected, the installation of stand-by tanks is commendable.

Were the flow curves for overflowing sewage and receiving river water similar, one might rely more confidently upon the transportation of suspended matter discharged from storm overflows to places of safe disposal down stream, but frequently during storms, the maximal discharge of suspended matter from the sewers occurs before the river has risen materially. Then, deposits occur which, once formed, are less easily transported down stream than is the component suspended matter before its deposit.

The stand-by basins also serve as emergency additions to the treatment works, by preventing excessive rates through the subsiding basins, and by feeding more gradually the load of suspended matter. On the other hand, is it not probable that the bulk of true domestic sewage solids is brought to the treatment works with the flows which the intercepting sewer will carry, and that the solids stored in the stand-by basins are largely those derived from the storm sewage and, therefore, are different in character?

The operation of these basins in practice would be an interesting subject for a subsequent paper because, like all works for the treatment of sewage, the method of their operation is a large factor for success or failure.

W. W. Horner,[16] M. Am. Soc. C. E. (by letter)[16a].—Apparently, because of the removal of the normal dry-weather flow from the Scioto River, the problem at Columbus, Ohio, approached quite nearly the drainage situation of those residential communities commonly traversed by smaller watercourses. To this extent, the situation has much in common with the sanitary and drainage problems now being encountered in connection with the improvement of the suburbs of some of the larger cities in this country, such, for example, as Westchester County, New York, sewerage, the suburban situation around Philadelphia, Pa., and Washington, D. C., and, more recently, the studies that the writer has had to make in connection with the proposed sewerage in St. Louis County, Missouri.

In many of these cases, the existence of combined sewers, or the belief that storm water may enter through cellarway drains or occasional downspout connections, makes it essential that storm drainage, to some extent, be allowed for in the main trunk or intercepting sewers. Because of this, these intercepting sewers might equally well be classified as deficient storm sewers and in common with the engineers in Columbus the writer has found it necessary to design on that basis.

In one large district a design was made on the basis of Fig. 16 for the preparation of which it was assumed that 50% of the houses would have downspouts connected to the sewers and that the gutter system for each house would be more than 50% efficient; actually, it was not expected that so many downspouts would become connected against regulations, but it was regarded as certain that many of the houses would have bell-traps at the foot of the out-

[16] Cons. Engr. St. Louis, Mo.

[16a] Received by the Secretary February 9, 1934.

side cellar stairs, and that an appreciable quantity of yard drainage would enter the system in that way.

The design was completed on this basis for the full lateral trunk system of a 10 000-acre district. In this instance, while it was proposed to permit overflow to the creek in one or two places, it was believed that the design was

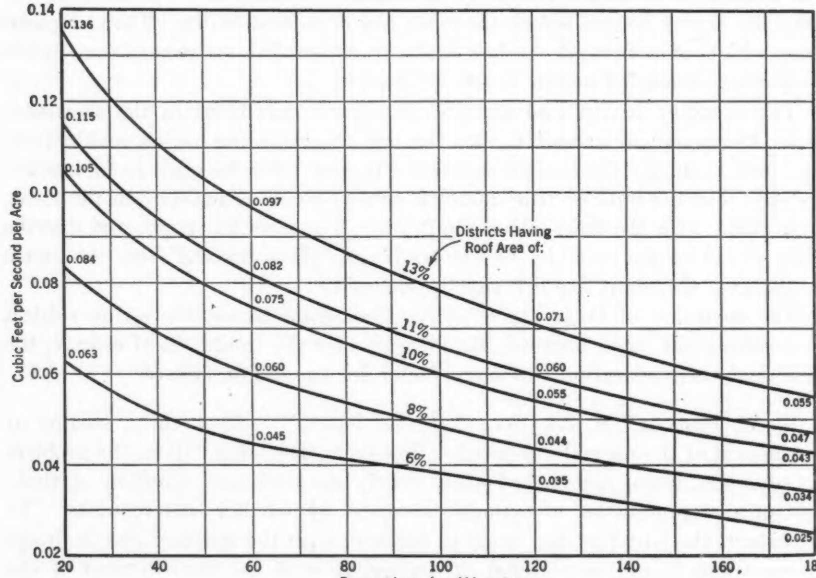

FIG. 16.—SANITARY RUN-OFF CURVES USED IN DESIGN OF SEWERS IN ST. LOUIS COUNTY, MISSOURI

actually balanced and that no overflow would take place. In another instance, the laterals were designed for a similar curve, but the intercepting, or more correctly, the main trunk, sewer paralleling the principal stream was designed on a basis of the maximum sewage flow being four times the average. Another condition was that this peak flow would be diluted with storm water on a 6 to 1 basis throughout the upper reaches, and on a 4 to 1 basis where it paralleled a larger stream before overflow occurred. In the latter case, a frequency of overflow was not determined accurately, but the probabilities would lie somewhere between four and six times per year. In this instance, the local stream was a small one and the dry-weather flow, exclusive of sewage, probably did not exceed 1 cu ft per sec.

In a third case, existing lateral sewers were laid out on a combined basis, and intercepting sewers were designed for six dilutions of the dry-weather flow without regulators. In general, the basis for sewer design in these districts fell into the following groups (the units being given in cubic feet per second, and the areas averaging 10 to 15 people per acre): (*a*) Sanitary sewers, designed for peak flow, about four times the average at 0.007; (*b*) intercepting

sewers, carrying four to six dilutions of storm water, about 0.04; (*c*) design for some downspout connections (see Fig. 16), depending on time of flow, 0.10 to 0.05; and (*d*) full storm-water design, depending on time of flow, 1.82 to 0.4. It appears that the Olentangy-Scioto Sewer carries six or seven dilutions of storm water.

The writer notes that, in computing storm-water run-off, the coefficient of 35% for rainfall is used for residential districts. Gaugings at St. Louis, Mo., indicate, in one residential block of fairly good slopes, a coefficient averaging 0.6 and, in one of much flatter slopes, a coefficient averaging slightly less than 0.4. For a semi-commercial block, the St. Louis coefficients will average slightly more than 0.8 as compared with the 0.75, the latter being a very satisfactory agreement.

In discussing this paper with other engineers, the question has been raised as to whether an estimate was prepared for a complete new system of sanitary sewers for the area now served by combined sewers, leaving the latter to act as storm sewers only, and whether the cost of such a new system might have been, in any way, comparable with the alternatives actually adopted. It would also be interesting to know whether such a suggestion might have been impracticable because of a fixed habit of connecting downspouts to the house sewer system.

The various types of sewer structures to meet particular conditions are usually interesting, but an explanation of the reason that led to the use of vitrified clay liner-plates (particularly for their use in the inverts of some sections and for the full circle in certain others), would be a valuable addition to the paper.

In a recent St. Louis project, involving a depth of flow varying from one-half full to full, for the handling of sewage, vitrified clay liners were used for the upper half of the circle exposed to condensation, but a high-grade concrete was relied on for the lower part where the flow was continual.

The use of the storm stand-by tanks offers an ingenious solution, but the economics of their injection into the design have not been presented as fully as the writer would have wished. It would be interesting to know the estimated cost of extending the intercepting sewer, full size, to the disposal plant, and whether somewhat less elaborate detention basins might not have been used at that location.

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DISCUSSIONS

SOME SOIL PRESSURE TESTS

Discussion

By Messrs. O. K. Froehlich, H. L. Thackwell, and Jacob Feld

Dr. Ing. O. K. Froehlich[6] (by letter)[6a].—When attempting to interpret qualitatively and, as far as possible, quantitatively, the results of soil pressure tests, dealt with in Mr. Parson's valuable paper, one must keep in mind that:

(1) The well-known, existing, so-called "classical" theories on earth pressures relate to a two-dimensional problem, such as an infinitely long retaining wall.

(2) The earth pressure problem of a sand or gravel fill, contained in the apparatus illustrated in Fig. 1, is three-dimensional.

(3) The bulkhead and side walls of the apparatus in Fig. 1 were not absolutely rigid, but were flexible to some extent, and the deflections were not measured.

(4) The deflections, caused by water only (no soil), did not influence the distribution of water pressures.

(5) The "dry" fills were not placed regularly in thin layers in the apparatus, but were shoveled in, so that the equilibrium of the lower portion probably was affected by dynamic influences.

(6) The measured values of the weight of material (loose and rodded) per cubic foot are only approximate average figures.

(7) The vertical wall friction was not measured.

(8) The difference between the quantity of water used for irrigation and the quantity that drained, was not measured.

Consequently, it seems to be advisable to avoid exaggerated mathematical rigidity in interpreting the most important results of these earth pressure tests. First, consider the simple case of an infinitely long vertical retaining wall with a height, h, and supporting a horizontal, cohesionless back-fill,

Note.—The paper by H. de B. Parsons, M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1934, by J. C. Meem, M. Am. Soc. C. E.; and February, 1934, by Messrs. Eugene E. Halmes, and L. C. Wilcoxen.

[6] The Hague, The Netherlands.

[6a] Received by the Secretary December 13, 1933.

weighing w units per unit of volume. Let ϕ be the angle of internal friction, and ϕ', the angle of friction between the wall and the fill. According to the general wedge theory, the horizontal component, H_a, of the active lateral earth pressure (on the assumption that $\phi = 40°$ and $\phi' = 37° \ 30'$) amounts to:

$$H_a = 0.207 \left(\tfrac{1}{2} \, w \, h^2\right) \cos \phi' = 0.164 \left(\tfrac{1}{2} \, w \, h^2\right) \ldots\ldots\ldots\ldots (1)$$

Leaving the inclination, ϕ', of the total earth pressure unchanged, the horizontal component, H_p, of the passive earth pressure is:

$$H_p = 1.11 \left(\tfrac{1}{2} \, w \, h^2\right) \cos \phi' = 0.880 \left(\tfrac{1}{2} \, w \, h^2\right) \ldots\ldots\ldots\ldots (2)$$

Active and passive pressures are created by slight wall movements away from, and toward, the fill, respectively. Between these values, H_a and H_p, there is a "neutral" earth pressure,[7] H_i, which corresponds to the motionless, or rigid, wall and amounts to:

$$H_i = 0.5 \left(\tfrac{1}{2} \, w \, h^2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The ratios between active and "neutral" and between passive and "neutral" pressure, according to Equations (1), (2), and (3), are, respectively:

$$\frac{H_a}{H_i} = 0.328 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and,

$$\frac{H_p}{H_i} = 1.76 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Assume that these ratios also hold true for the three-dimensional problem of the bin, having flexible side walls. At present, this assumption can only

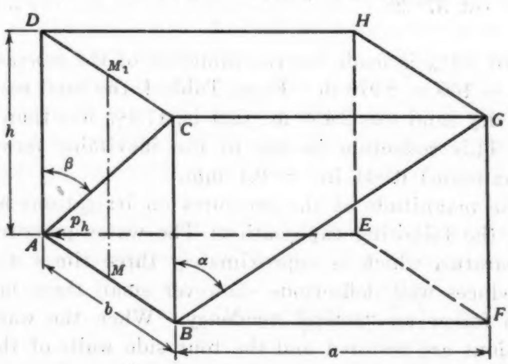

FIG. 9.—COMPUTATION OF "NEUTRAL" EARTH PRESSURE

be checked empirically, as the three-dimensional general wedge theory has not yet been developed.

[7] "Drukverdeeling in Bouwgrond," *De Ingenieur*, 1932, No. 16; 1933, No. 25.

The theory of the three-dimensional "neutral" earth pressure problem[8] enables one to determine the lateral pressure, exerted by the granular fill of a rectangular bin having a section of 7.0 by 5.0 ft, and a height of 7.0 ft, if the side walls and bottom of this bin are practically rigid.

The horizontal pressure, p_h, per unit of area at Point A of the side wall, $b \times h$, is approximately given by the following expression (see Fig. 9):

$$p_h = \frac{w\,b}{2\,\pi}\left\{\log \cot \frac{\beta}{2} \tan \frac{\alpha}{2} + \cos \alpha\right\} \cot \phi' \quad \ldots\ldots\ldots\ldots (6)$$

By applying Equation (6) to the points of the lines, $A\ D$ and $M\ M_1$, Fig. 9, at depths of 1, 2, 3, ..., 7 ft, under the surface, $C\ D\ H\ G$, of the fill in the bin, the unit pressures, p_h, in Table 6, are obtained by multiplying each value by $\frac{wb}{2\pi} \cot \phi'$. The averages in Table 6 are obtained, as follows (at a depth of 1 ft, for example: $0.199 \times \frac{2}{3}(0.390 - 0.199) = 0.327$).

TABLE 6.—"NEUTRAL" UNIT PRESSURES, p_h, ON BULKHEAD

Depth, in feet	Side line, $A\ D$	Center line, $M\ M_1$	Average
1	0,199	0,390	0.327
2	0.383	0.726	0.611
3	0.547	0.995	0.844
4	0.685	1.202	1.029
5	0.799	1.363	1.175
6	0.889	1.481	1.283
7	0.960	1.578	1.372
Total	....	....	6.641

Using the values, $w = 101.5$ lb per cu ft for sand; $b = 5$ ft; and $\phi' = 37°\ 30'$; the total thrust is:

$$\frac{101.5 \times 5}{2\,\pi} \cot 37°\,30' \,\frac{(2 \times 6.641 - 1.372) \times 1 \times 5}{2} = 3\,130 \text{ lb}$$

If an allowance of 5.1% is made for the influence of the canvas at the edges, etc., $H_t = 3\,130 - 160 = 2\,970$ lb. From Table 4, the total scale pressure on the bulkhead for dry sand was 2 453 lb; that is, 17.4% less than the "neutral" earth pressure. This reduction is due to the inevitable movement of the bulkhead of (maximum) 0.004 in. = 0.1 mm.

As regards the magnitude of the pressures on irrigations and drainages, the writer offers the following explanation: The water pressure on the side walls of the apparatus, which is approximately three times as great as the earth thrust, produces wall deflections—however small these may be — and allows the fill to follow an "active" tendency. When the water is drained away, the deflections are restored and the four side walls of the bin exert a passive pressure on the fill.

To develop this idea, one must keep in mind: First, that the unit weight of the sand immersed in water was 61% of the weight in air; and, second, that the principle of superposition of pressures must remain valid in the present case.

[8] "Drukverdeeling in silo's * * *", *Polytechnisch Weekblad*, 1933, Nos. 38–39.

According to Equation (4), the sand pressure against the bulkhead, if the sand is saturated, must be:

$$H_a = 0.328 \times 0.61 \times 2\,970 \ldots\ldots\ldots = 594 \text{ lb}$$

Water pressure according to Table 4.......... $= 7\,080$ lb

Total.................. $= 7\,674$ lb

Table 4 gives 7 691 lb as the average result. After drainage the sand pressure against the bulkhead is passive, and Equation (5) yields: $H_p = 1.76 \times 2\,970 = 5\,230$ lb. Table 4 gives 5 052 lb as the average result.

These figures would confirm the previously mentioned opinion, that slight deflections of the side walls took place during irrigation and were restored during drainage. In an absolutely rigid apparatus the following total pressures might be expected: Dry, 2 970 lb; saturated, $0.61 \times 2\,970 + 7\,080 = 8\,890$ lb; and, drained, 2 970 lb.

H. L. THACKWELL,[9] ASSOC. M. AM. SOC. C. E. (by letter)[9a].—Due to the paucity of reliable information on a subject that deals with the forces exerted on non-homogeneous materials, such as earth, sands, clays, and gravels, and a combination of all such materials under various conditions of saturation, the average practicing engineer has resorted either to personal judgment, or to "rule-of-thumb" methods, for assigning the forces assumed to act against walls, the sides of tanks, and bins. This excellent paper describing soils tested under saturated and drained conditions, with measured lateral pressures, is a valuable addition to a subject in which many engineers have been groping for a method of rational analysis.

If the conditions of loading, and the physical properties of such non-homogeneous, granular, and plastic materials, could be ascertained accurately, it would be possible to develop a correct theory for the particular case under consideration. Correct theory is only possible when based on inductive reasoning (from the particular to the general); the particulars must be studied by experiment, and such experiments must be based on conditions that are likely to be encountered in practice. The classical theories of Rankine and Coulomb have served very well in the design of walls back-filled with dry materials of uniform nature. With these formulas, ϕ is the angle of repose, although Boussinesq has pointed out that the angle of internal friction would be more applicable.

In the frequent case of a wall to be designed with a vertical back and with the requirement that it retain a level bank of earth, the customary formula is:

$$P = \frac{w}{2} h^2 \tan^2 \left(45^\circ - \frac{\phi}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

If this formula is correct in theory, it is necessary, in order to use it properly, to assign the correct values for w and ϕ, which are the only unknowns. If the conditions of back-filling are known in advance, it would be possible, by

[9] Cons. and Const. Engr., Tyler, Tex.

[9a] Received by the Secretary January 15, 1934.

experiment, to obtain the weight of the material, or combination of materials, to be used in the fill, and also the angle, ϕ, which should be the angle of internal friction.

In practice, it is seldom that the engineer who is entrusted with the problem of designing simple concrete retaining walls, will have the time or the money to make the necessary experiments for determining the correct values of the aforementioned variables. For example, consider that an engineer has been retained to design, and supervise the construction of, a sewage disposal plant for a city of 5 000 population. The usual procedure is to select a suitable site for the plant, and then to make a topographic map of it, including a few borings for soil and sub-surface drainage conditions. This map, with the borings for a guide, then becomes the field for office locations of all tanks, walls, etc., that will be incorporated in the design. The designing engineer, after a few trial layouts, will finally select the exact positions for the various buried tanks and the retaining walls for filter enclosures. It is impractical to expect that the engineer entrusted with the design of a small project, and receiving only a 5% fee for engineering work, will make a sufficient number of tests of the filling material and of the angles of internal friction, which will occur at various positions behind the structures, in order to obtain accurate values that will eliminate the unknowns of the equation.

Moreover, the conditions after back-filling are subject to change, due to a rise in the ground-water table, rises and falls in temperature, freezing of the surface layers, or to the peculiar properties of certain (black land) clays which, on drying out, shrink away from the surface of structures.

The usual methods of back-filling around tank walls and filter retaining walls, are to dump the previously excavated material, reserved for that purpose, into the excavation and puddle it with water. Under these conditions the materials have become more or less mixed in the process of excavating and removal for storage, which storage pile is later used for back-fill and probably removed some distance from the locality where it was excavated.

With all these conditions in mind, and with "post" reflections on completed walls, the writer, against engineering approval, has been guilty of using the unscientific method of "equivalent fluid pressure" for approximating earth thrusts on minor walls and structures. With the "equivalent fluid method," of course, considerable is left to the judgment of the engineer. Table 7 is offered in an attempt to justify the "fluid method" of approximate thrusts, using Rankine's formula for the bouyant earth pressures under complete saturation, and considering the water pressure added to the lateral earth pressures, ϕ being the angle of internal friction of the material. The head used is 6.979 ft as in the soil test apparatus of the paper, the angle, ϕ, being 58° for the lightest material and increasing 1° for each 10 lb of weight, until a maximum of 63° is reached for a weight of 120 lb of dry material. The specific gravity was assumed as 2.60, and the weight of water taken as 62.22 lb per cu ft.

It can be seen from Table 7 that, in a fully saturated soil, the resultant combined lateral pressures of earth and water are practically the same for all possible weights of soil. It is quite logical then to assume that an equivalent

fluid pressure may be adopted, which will give the same results as the more refined and exact method, provided the angle of internal friction has only slight variations with different weights of soil, which are likely to become incorporated in the fill.

TABLE 7.—EFFECT OF VARIOUS WEIGHTS OF SATURATED SOILS ON LATERAL PRESSURES, USING RANKINE'S FORMULA WITH A VARIABLE VALUE OF ϕ.

Description	Dry weight of material, in pounds	Void in soil, percentage	Weight of contained water, in pounds	Combined weight under saturation, in pounds	Weight of earth in water, in pounds	$\frac{w}{2}$	$\tan^2$	h^2	p, due to earth pressure	P, due to water pressure	Total lateral pressures
Loose fine sand..	80	50.6	31.5	111.5	30.7	15.35	0.082	48.75	61.4	1 510	1 571.4
Coarse sand....	90	44.4	27.6	117.6	34.6	17.3	0.077	48.75	64.9	1.510	1 574.9
Graded sand....	100	38.3	23.8	123.8	38.4	19.2	0.072	48.75	67.4	1 510	1 577.4
Packed sand....	110	32.1	20.0	130.0	42.2	21.1	0.067	48.75	68.0	1 510	1 578.0
Packed sand and gravel........	120	25.9	16.1	136.1	46.1	23.05	0.062	48.75	69.8	1 510	1 579.8

In this case of full saturation the equivalent fluid would have a weight of 66 lb per cu ft, and with a drained condition would have a weight of 44.2 lb per cu ft, this latter pressure being obtained by using the corrected total pressure of 1 077 lb as shown in Column (6), Table 4. The writer has been using a value of 40 lb for drained conditions, and 70 lb for a full fluid condition of back-fill, for East Texas sands and clays.

The phenomenon illustrated in Fig. 3 of decreasing pressures as the water rises up to the 1-ft mark, may be due to air binding. Water rising under a sand filter would have the effect of compressing air in the voids of the sand, and until this compressed air reached a pressure sufficient to overcome the frictional resistance of the sand grains it would not be released Under the condition of partial air compression, some uplift would occur which would be reflected in the lateral pressures as measured.

The writer agrees with the author in his "Summary" of conclusions, and is grateful for this valuable addition to the meager information on this subject, now at the disposal of practicing engineers.

JACOB FELD,[10] ASSOC. M. AM. SOC. C. E. (by letter)[10a].—The description of tests to determine the lateral pressure of sand and gravel with various degrees of saturation, as offered by Mr. Parsons, is clear and complete. To permit comparison with the paper and with other tests, the present remarks are divided into discussion of the apparatus, results, and conclusions.

Apparatus.—It is unfortunate that the apparatus was not equipped with provision for measuring the vertical components. Since the pressure of the water on the vertical test wall is horizontal, the effect of the soil in water at varying heights would have been more striking. The calibration of the apparatus with water cannot be questioned. It was found that the effective width of the wall for water was 4.75 ft, and it is assumed that the same

[10] Cons. Engr., New York, N. Y.

[10a] Received by the Secretary February 1, 1934.

applies for the soils. Such assumption cannot be accepted without further proof, especially because of the converging sides of the bin. Such convergence has no effect on water pressure, but must have a decreasing effect on soil pressure. The volume of the wedge of rupture (actual or fictitious) is reduced. The pressures on the end vertical strips of the test wall are very much less than those on the inner strips. A further test can give some valuable data; namely, to add soil to a bin full of water and determine the increase in pressure under these conditions. In this type of test, the variation in the vertical component would give important information.

Results.—The material used is described as a bank sand and gravel. The effect of water on this material is striking. When water was added, complete saturation could not occur immediately. A time interval must pass before the entrapped air is expelled. During this period the pressure decreased—the amount of decrease being less for gravel than for sand. In spite of a smaller percentage of voids, the individual interstices in the gravel were larger than in the sand, and the air could leave faster. Without complete saturation and a continuous film of water, hydrostatic pressure could not be recorded. Apparently, additional moisture content under partial saturation decreased the pressure because of an increase in the coefficient of internal resistance of the material.

Experiments reported by the writer[11] on sand containing from 5 to 9% moisture indicated a uniform relationship between the coefficient of internal resistance and lateral pressure. From the amount of shrinkage of loose fill (see "Pressure Tests"), the sand used in these tests could hold a considerable percentage of water. In the compacted state, as a result of flooding or ramming, the coefficient of resistance will be greater than in the loose state.

For the loose sand, the author reports a value of 40° for the angle of internal resistance, or a coefficient of 0.839. An increase of this coefficient by less than 10% to 1.00, or 45°, would mean a decrease in lateral pressure of $22\frac{1}{2}\%$ (theoretically).

The low values for the position of the resultant pressure are caused partly by the shape of the bin, eliminating larger parts of the wedge of rupture in the upper levels than in the lower levels.

Conclusions.—The most important conclusion is that, the belief of many "practical" engineers notwithstanding, a lateral pressure greater than the hydrostatic pressure of water can occur.

The "arching" effect was entirely disproved by the tests at the University of Cincinnati[11] where identical results were obtained with rigid and free side walls. The discussion of theoretical and practical reasons for the elimination of the angle of repose from any consideration is also included in the paper reporting these tests.[11]

The point of application of the resultant pressure for consolidated fills should be above the one-third point because consolidation is equivalent to surcharge. The effects of saturation and drainage must cause internal re-adjustments in the fill—more so in sand than in gravel, with greater variation in results from theoretical values for sand than for gravel.

[11] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PRACTICAL RIVER LABORATORY HYDRAULICS

Discussion

BY MESSRS. I. H. PATTY, CHARLES S. BENNETT, AND KENNETH C. REYNOLDS

I. H. PATTY,[21] ESQ. (by letter)[21a].—In this excellent paper Lieut. Vogel presents Equation (12) as a time scale for models, stating that this formula is applicable to reservoir problems, or to flood-control problems where the time element is important, such as in the filling of back-water basins and other storage areas. The author further states that the formula is not applicable in problems involving bed-movement development; that no formulas have been devised which reliably indicate the relationship for such development; and that such ratios must be determined by actual tests.

During the short space of time that has elapsed since the establishment of the U. S. Waterways Experiment Station, distinct advances have been made in the technique of designing and operating hydraulic models. This is particularly true in connection with experiments on models of the movable bed type. Such models have beds moulded of sand or other transportable material and are used when it is desired to determine the effects of a stream on its bed and banks. They are useful in problems involving river regulation where it is desired to study effects of dikes, channel alignment, methods of increasing depths for navigation, effects of cut-offs, caving banks, and general channel stabilization. The technique of experimentation with these models has advanced to the stage that quantitative results are regularly attained with a fair degree of accuracy. Naturally, the value of the findings are increased enormously if the time of their accomplishment in Nature can be predicted. To make such predictions requires a scale value indicating the relationship between the time required to produce given effects in the model and corresponding effects in the prototype.

There are two methods whereby a time scale for a movable bed model may be developed. A time scale may be established when at least two surveys,

NOTE.—The paper by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1934, by Messrs. Lorenz G. Straub, Paul W. Thompson, Ralph W. Powell, K. D. Nichols, and Frank W. Edwards.

[21] Asst. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[21a] Received by the Secretary January 11, 1934.

taken at different times, are available of the territory represented by the model. The procedure is to determine by actual observation the length of time required for the model to reproduce known changes that have occurred in its prototype. The scale is then established as the ratio between the observed time required by the model and the prototype to effect these changes. This is the method that has been most commonly followed by experimenters in the past, and it is fairly accurate where the necessary survey data are available.

For many years attempts have been made to devise time scales by straight mathematical procedure. Such formulas usually have had factors representing the relationships for length, depth, slope, and mean grain diameter of bed materials. When applied to models for which scales have been derived experimentally, they have invariably indicated relationships so widely at variance, and so conflicting, that they have come to be considered, not only as having little value, but as being misleading.

A formula for a time scale has been derived, mathematically, and has been tested on the movable bed models built at the U. S. Waterways Experiment Station. On several of these models, experimentally derived time scales were also developed, and were found to agree with those determined by the theoretical method, indicating the latter to be both accurate and dependable.

A stream of water (either a model stream, or its prototype) performs work and loses energy as it flows down-hill, transporting its burden of sedimentary matter, and shifting the materials of its bed to change its hydrographic features. The ability of a movable bed model to simulate the hydrographic configurations of its prototype was demonstrated more than fifty years ago by Fargue, and such results are now regarded as routine technique in hydraulic laboratory practice. Scales for such characteristics in the model as length, depth, slope, velocity, discharge, etc., are derived by direct linear relationships. The time scale developed at the Vicksburg Laboratory after Lieut. Vogel's paper was written, shows the ratio between the time required for a model and its prototype to perform similar work.

Work is done by a force moving a body through any distance. The product of the numerical value of the force and the distance is a measure of the work, or it may be expressed by the equation:

$$W = F\,L \qquad (59)$$

in which, W = work; F = force, and L = length traveled. Equation (59) may be adapted to express the work done by a flowing stream. It was shown by du Boys, in 1879, that the tractive force of flowing water may be expressed by the formula:

$$J\,D\,S \qquad (60)$$

in which, D = depth; S = slope; and J = the weight of a unit volume of the liquid. Placing $J\,D\,S$ equal to F in Equation (59):

$$W = J\,D\,S\,L \qquad (61)$$

Similarly, a work scale, W, denoting the ratio between the work done by a model and its prototype per similar unit length, can be derived if the respective values for the model are divided by those for the full scale:

$$\frac{W_m}{W_n} = \frac{J_m\, L_m\, D_m\, S_m}{J_n\, L_n\, D_n\, S_n} = l\, d\, s = w \quad \ldots\ldots\ldots\ldots\ldots (62)$$

The scale, $l\ d\ s$, would also denote the ratio between the time required for the model and the prototype to perform similar work if the velocity of flow in each were the same. This is never true in river models, but if the work scale, $l\ d\ s$, is divided by the velocity scale, v, the result will be a unit-time-work scale, t_w:

$$t_w = \frac{l\, d\, s}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

From Manning's formula the scale value for velocity in models has been found about equal to $d^{\frac{2}{3}}\, s^{\frac{1}{2}}$, the expression, $\frac{1.486}{n}$, being cancelled since it is quite nearly equal to unity. In Equation (63), let $v = d^{\frac{2}{3}}\, s^{\frac{1}{2}}$; then:

$$t_w = \frac{l\, d\, s}{v} = \frac{l\, d\, s}{d^{\frac{2}{3}}\, s^{\frac{1}{2}}} = l\, d^{\frac{1}{3}}\, s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots (64)$$

Equation (64) is a complete expression for a unit-time-work scale, showing the ratio between the time required for a model and its prototype to perform similar work. It presumes the bed material in each to be identical, or nearly so, and when they are dissimilar a factor, f, denoting the relative transportability of the two materials, as determined by one of several existing formulas or by flume test, must be added and the scale computed for each of the stage heights at which the model is operated. The complete formula, then, will be:

$$t_w = f\, l\, d^{\frac{1}{3}}\, s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

The mathematical soundness of Equation (64) may be further demonstrated by developing it from a different angle. A tractive force scale, $d\ s$, may be derived as follows:

$$d\, s = \frac{J_m\, D_m\, S_m}{J_n\, D_n\, S_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

in which, the factor, J, being equal to unity, cancels out. A unit-time-distance scale, t_d, showing the ratio between the distance traveled in the model and in Nature, in unit time, may be derived by dividing the length scale, l, by the velocity scale, v, or,

$$t_d = \frac{l}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

Then the product of the unit-time-distance scale, $\frac{l}{v}$, and the tractive force scale, $d\ s$, will give the unit-time-work scale, or,

$$t_w = \frac{l}{v}\, d\, s = \frac{l\, d\, s}{v} = l\, d^{\frac{1}{3}}\, s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots (68)$$

Most of the movable bed models constructed at the U. S. Waterways Experiment Station have horizontal scales (l) of between 1:1 000 and 1:500, and vertical scales (d) of from 1:150 to 1:100. On such models the average time of operation, indicated to simulate 1 year in Nature, is from 6 to 16 hours.

Experience gained on nearly one hundred model studies has demonstrated the reliability of the various formulas given by Lieut. Vogel in his paper. With the exception of a formula for determining the time relationship for movable bed models they comprise all the scale values commonly needed in a river hydraulics laboratory. Other methods of deriving a time scale for such models are being investigated.

Charles S. Bennett,[22] M. Am. Soc. C. E. (by letter)[22a].—Engineers engaged in the field of applied hydraulics should welcome this excellent compilation of data on laboratory methods of solving numerous hydraulic problems. The use of models in the study of certain hydraulics phenomena furnishes the only practical and reliable solution to many problems confronted by designers of hydraulic structures.

The writer wishes to call attention to the fact that expensive equipment and elaborate laboratories are not always necessary for the application of some of the studies outlined by Lieut. Vogel. It is often possible to secure valuable data on specific problems by the use of a small amount of equipment and some ingenuity.

The improved channel of the Miami River at Dayton, Ohio, is designed to carry a regulated maximum flood flow with a free-board of 3 ft on the levees flanking the channel. Several years ago it was proposed to erect several low overflow dams in this channel to provide slack water during periods of low flow, primarily to enhance the appearance of the stream and to provide facilities for boating and swimming. Before such structures could be approved it was necessary to determine what effect they might have on river stages at times of high flow. As various shapes and types of dams were under consideration it did not seem feasible or safe to rely entirely on calculations of the effects of such structures.

It was decided to construct a small model of the section of the stream involved and to measure the effects of various types of dams, using flows corresponding to great floods. The reach of river channel to be investigated was about 6 000 ft long, with an average width of about 600 ft. Space available for the model limited the scale to 1:600 horizontal and 1:120 vertical.

A specially constructed table top of laminated wood, about 4 by 10 ft, was mounted on trestles set up on a concrete garage floor. The table was carefully and accurately leveled and adjusted with clamps so as to maintain its level position. The model was constructed on this table, using the top surface as a datum plane. Templates made of cardboard coated with shellac and spaced 200 ft apart were used. The model was made of a concrete consisting of cement and commercial asbestos in about equal proportions. This material

[22] Engr., The Miami Conservancy Dist., Dayton, Ohio.

[22a] Received by the Secretary February 5, 1934.

can be cut with a knife, nails can be driven in it, and it is relatively light in weight. Two paintings with a liquid water-proofing material made it impervious.

Water was furnished by a 2-in. direct-connected centrifugal pump, discharging into a baffled weir box in which was a 90° V-notch of steel plate. Water flowing through the channel passed out at the lower end under an adjustable tail-gate and emptied into a storage trough under the table, from which it flowed back to the pump. The maximum quantity of flow required was about 0.15 sec-ft.

Measurements of surface elevations in the channel were made with point gauges, each mounted on a beam spanning the channel. These cross-beams slid on long steel-bar beams mounted parallel to the channel on each side. The point gauges were equipped with verniers and could be read to 0.0005 ft. During tests an engineer's level was used to check all adjustable parts of the model set-up to insure accuracy.

Models of dams were cast of babbitt metal to fit the bed of the stream at pre-determined points. The various models could be interchanged as required.

The cost of this model was relatively small. The pump and motor cost $60 (second-hand) and were available for other uses. The total cost of materials was about $85. All labor was performed by regular employees who were handy with tools. Even the point gauges were "home-made," the verniers being salvaged from discarded level rods. Many valuable data were secured by the use of the model and most of the material was salvaged for subsequent experiments.

KENNETH C. REYNOLDS,[23] ASSOC. M. AM. SOC. C. E. (by letter)[23a].—The U. S. Waterways Experiment Station is carrying on excellent researches and the presentation of methods and equipment which have proved to be satisfactory at this Station, is of inestimable value to other experimenters in the field of river hydraulics. The author is to be commended for this excellent paper.

The writer agrees as to the desirability of having uniform nomenclature although he can not agree with some notations used by the author. If some Greek letters have been frequently used for certain factors, they might well be considered as standard. Thus, ν is used by many for the kinematic coefficient of viscosity.

Immediately preceding Equation (5), consideration is given to an undistorted model, and it is shown that the ratio of discharges is, $q = \frac{a\, r^{\frac{2}{3}}\, s^{\frac{1}{2}}}{n}$. In deducing Equation (5) it is assumed that the hydraulic radius is equal to the mean depth. This assumption is unnecessary since the hydraulic radius is an area divided by a wetted perimeter or is a linear dimension for an undistorted model. The ratio of these linear dimensions is l, from which Equation (5) follows.

[23] Asst. Prof. of Hydraulics, Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[23a] Received by the Secretary February 3, 1934.

In the paragraph preceding Equation (10) Lieut. Vogel states that it has been found from experiments with distorted models of rivers that the ratio of the roughness coefficients for model and prototype is unity, both for models built of cement mortar, if the scale ratio of length is less than 1 to 1 000, and also, for models with a sand bed when the scale length is between 1 to 1 000 and 1 to 500. This information is of real value. In 1925, Erik G. W. Lindquist, M. Am. Soc. C. E., determined[8] this ratio of coefficients analytically for an undistorted model with non-uniform flow to be the sixth root of the scale of length.

[8] "Om Modellregler Eller Likformighetssatser vid Vattenbyggnadstekniska Försök," by E. G. W. Lindquist, *Teknisk Tidskrift*, Stockholm, 1925, Nos. 30, 34, and 43.

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DISCUSSIONS

FORMATION OF FLOC BY FERRIC COAGULANTS

Discussion

BY MESSRS. EDWARD S. HOPKINS, W. D. HATFIELD, L. B. MILLER, AND LINN H. ENSLOW

EDWARD S. HOPKINS,[26] ESQ. (by letter)[26a].—Practically applied, this paper shows the necessity of completely dispersing and oxidizing the iron salt before the addition of the alkali to produce floc. A fairly high velocity (1 ft to 2 ft per sec) of mixing must be maintained for complete dispersion, followed by a short period (5 min) of agitation to oxidize the iron by aeration. Upon the completion of this initial mixing the alkali should be added to form floc. Upon addition of the alkali a colloidal iron solution is immediately formed and by continued mixing at high velocity, coagulation occurs. A decreasing velocity through the basin beyond this point will produce a compact floc of greater tenacity than if uniform speed is maintained. Such a floc settles readily. This is an important factor in mixing-basin design and should be considered in future plans. It is believed that trouble with "pin-head" floc can be frequently traced to insufficient velocity or time of mixing. Microscopic examinations indicate that the physical properties of floc depend in a large measure on mixing conditions.

Coagulation of color in waters is a different problem chemically from that of turbidity. The sulfate ion, obtained from either alum or ferrous sulfate, is an important component of the floc and is of equal value to the iron or aluminum ion for coagulation of turbid waters. Color combines only with the aluminum or iron ion forming the flocculated organic particles of color floc. This coagulation is best at very low pH-values (4.0 to 5.0). When alum is used, it is becoming good operation practice to settle the color floc, then add coagulant and alkali, thereby raising the pH, with additional mixing to coagulate the turbidity followed by secondary sedimentation, the final effluent passing to the filters. The data presented in this paper, together with information from the Montebello Laboratory, indicate that if the color

NOTE.—The paper by Edward Bartow, M. Am. Soc. C. E., and Messrs. A. P. Black, and Walter E. Sansbury, was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 19, 1933, and published in December, 1933, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[26] Prin. San. Chemist, Montebello Filters, Baltimore, Md.

[26a] Received by the Secretary January 16, 1934.

is coagulated by iron salts in the initial part of the mixing basin, alkali may be added in the latter part, necessitating only one sedimentation period. Analysis of floc thus obtained gives a formula of $Fe_2O_3 \cdot 3H_2O \cdot 2SiO_2 \cdot 4X$, when X represents the organic color material in combination. These results indicate that iron salts may be more advantageous for color coagulation than alum.

This study indicates that coagulation with iron salts reaches a maximum between pH-values of 6.0 to 6.8. As stated by the author, it is at variance with previous work done in the Montebello Laboratory. His assumption that insufficient oxidation of these salts explains this difference appears to be correct and should be accepted unless future work reverses it. Floc formation with chlorinated copperas showed clearly that when the iron was completely oxidized by chlorine, precipitation occurred at very low pH-values (4.0). Iron and lime coagulation is utilized in the Baltimore plant to remove manganese from the supply. This element is not adsorbed by this treatment below a pH-value of 9.0; therefore, no contemporary data are available relating to the effect of oxidation at lower values.

By presenting fundamental data of this type to a group of engineers, Professor Bartow has shown clearly the necessity of trained chemical personnel in charge of plant operation. This is particularly true for the small water plant; the large ones have such personnel. An excellent junior engineer in charge of plant construction is frequently given its operation after completion, with inefficient results. Under such circumstances the individual is not at fault; it is the wrong type of training that he possesses. Coagulation control is a highly complex problem in colloidal chemistry; therefore, operators should be fundamentally trained in this science thereby giving them a better understanding of the problems involved. Considerable improvement in operation would also result if a closer co-operation could be secured between the designing engineer and the chemical engineer. As is well known, the characteristics of every water are different and, therefore, each source of supply, apart from its engineering problem, is a chemical problem. The advice of the chemical engineer relating to details of plant design would be of mutual benefit and would assure a more smoothly operated plant than is now found in many localities. The architect is needed to provide a beautiful plant, the chemical engineer to minimize operating troubles.

W. D. Hatfield,[27] Assoc. M. Am. Soc. C. E. (by letter)[27a].—Until recently the coagulation of natural waters by means of iron, lime, and aluminum salts, before sedimentation and filtration, has been more of an art than a science. Although a well-trained chemist was best fitted to study the idiosyncracies of water treatment, even he had to use "cut-and-try" methods which, combined with hard-earned experience, would produce a satisfactory drinking water with a minimum of color, odor, and taste. Each water has been a distinct problem (some more than others) and will remain a distinct problem for some time to come. However, in recent years, material advances have been

[27] Supt., Sewage Disposal Plant, Decatur, Ill.

[27a] Received by the Secretary January 17, 1934.

made in the knowledge and theory of inter-ionic activities. R. S. Weston, M. Am. Soc. C. E., first used pre-chlorination to aid in color removal. Soon afterward numerous investigators began studying the important action of the concentration of hydrogen ions on iron and aluminum flocs and color colloids. For a while the pH, or hydrogen-ion concentration, was believed to be the true governing factor in coagulation, but, later, this guide was found to vary with different types of water; in some cases the pH-zone of coagulation was narrow; in others, it was wide; in some cases, the optimum pH for coagulation was 6.3, in others it was 5.3, and in some water it was as low as 4. In the latter case iron had to be used as the coagulant because alum would not coagulate satisfactorily at a pH of 4.0. When one coagulant would not work another, and then another, was tried.

The individuality of a natural water is due to the compounds it contains in solution, and to the equilibrium between these compounds and the coagulant added. This phase of water treatment has been sadly neglected. The authors have made a real contribution to this field, by showing the effect of the chloride, sulfate, sodium, and calcium ions in conjunction with hydrogen and hydroxyl-ions on the coagulation of the ferric-ion. Such contributions as these will soon completely remove water purification (coagulation) from an art to a science which is based on a definite knowledge of the part played by each individual component of the solution.

The use of ferric salts in water treatment followed its use by Messrs. F. W. Mohlman and J. R. Palmer in the coagulation of activated sludge in sewage treatment. The practical problems in water, sewage, and sludge treatment differ greatly, but fundamentally the theory on which coagulation of iron salts rests, must be basic and applicable in both fields. It is hoped that the authors will carry to a conclusion these ionic studies, using all the ions found in waters, sewages, and sludge liquors. Perhaps even the sewage field may some day rest on a firmer theoretical basis, due to a clearer knowledge of these ionic activities.

L. B. Miller,[28] Esq. (by letter)[28a].—In addition to summarizing clearly the pertinent work previously done on ferric coagulants for water clarification, its significance, and limitations, the authors have made a most significant contribution to the theory and practice of the use of ferric coagulants. The zone of non-floc formation between the pH-values of approximately 7.0 to 8.0 could not be anticipated; and, indeed, in some of the earlier work in which more concentrated solutions were used than those by the authors, it remained unnoted. The importance of this zone of non-floc formation in the practical application of ferric coagulants to water clarification cannot be over-estimated. It indicates clearly the conditions under which the water-works engineer must operate, in order to achieve optimum results.

The theory of the use of additional salts, such as those of alkali and lime, to control partly, the formation of floc in the zones in which ferric coagulants exhibit a reluctance to floc formation, is interesting, and the practical signifi-

[28] With Johns-Manville Research Laboratories, Manville, N. J.

[28a] Received by the Secretary January 18, 1934.

cance of the application of this theory may be of considerable importance in certain cases.

The comparison of alum with ferric coagulants in the clarification of waters highly colored by organic matter, where clarification must be carried out at low pH-values, the advantages, and limitations of each of these types of coagulants, are likewise very opportunely emphasized.

The authors are to be congratulated upon this excellent contribution to the theory and practice of water clarification by ferric coagulants.

LINN H. ENSLOW,[29] ASSOC. M. AM. SOC. C. E. (by letter)[29a].—In Nature a certain amount of natural coagulation is constantly going on, in which iron salts play an important rôle. Soluble iron is contributed through reducing reactions on iron compounds in the rocks and soil. Later, upon becoming oxidized, the iron thus brought into solution flocculates in running streams and at or near the surface of water bodies at rest. It is highly probable that such natural coagulation is responsible, to a considerable extent, for the gradual lessening of organic color in new reservoirs, more particularly the deep ones. In the latter the same iron is returned to the solution stage after having settled to the bottom and is released to the upper strata to flocculate anew. Diffusion, wind action, and turnover of reservoir contents due to wind action and temperature changes, serve to complete the precipitation-solution-precipitation cycles.

Whenever the writer notes references in the literature that laud the work of Mr. L. B. Miller, who in 1923 to 1925 studied coagulation effects and the composition of the floc produced under varying conditions, he is always hopeful that some one will look a little farther back and give some of the deserved recognition to an earlier worker, Dr. Frank E. Hale, who did not have the new "constellation," pH, to serve as the remarkable tool that it is in coagulation studies and control. Hale, working with various waters, reached the conclusion that color removal was due primarily to the formation of a complex compound from the acidic color compound and the amphoteric hydrate of alumina. He explained why effective flocculation and precipitation did not necessitate the theoretical quantities of alkali in certain waters, and his paper[30] abounds in theoretical discussion and conclusions which have been substantiated in practice or confirmed by subsequent investigators—but rarely, if ever, with acknowledgment.

Early Work with Chlorinated Copperas.—The origin of chlorinated ferrous sulfate (chlorinated copperas) can be traced to the laboratories of the Chicago, Ill., Sanitary District. Ferric chloride had proved considerably superior to alum and other coagulants for conditioning sewage sludge for dewatering. Its "drug store" price was the chief obstacle to its use. The simple expedient of oxidizing ferrous sulfate solutions to produce a mixture of ferric sulfate and

[29] San. Engr., The Chlorine Inst., Inc.; Editor, *Water Works and Sewerage*, New York, N. Y.

[29a] Received by the Secretary January 12, 1934.

[30] "The Relation Between Aluminum Sulphate and Color in Mechanical Filtration," by Dr. Frank E. Hale, *Journal of Industrial and Engineering Chemistry*, Vol. 6, August, 1914, p. 632.

ferric chloride was the outgrowth of a search for the most inexpensive ferric coagulant obtainable. Its efficiency was lower than that of ferric chloride, but the over-all cost to obtain desired effects was sufficiently less to cause its adoption at Houston, Tex., for activated sludge dewatering. Shortly afterward, the City of Milwaukee, Wis., followed in quick order and although chlorinated copperas can still be used at a lower cost at Houston, Milwaukee found it less costly to purchase, on long-time contract, a supply of ferric chloride in solution which is shipped in rubber-lined tank cars. Since then several sewage plants—notably Gastonia, High Point, and Charlotte, N. C., and Pasadena, Calif.—have supplanted other conditioning agents with ferric chloride.

Coagulation of Water.—Following the successful adaptation of chlorinated copperas to sludge dewatering, the writer made several tests of its use as a coagulant for water and industrial wastes. It was soon apparent that it was a coagulant considerably superior to others in the low pH range (5.5 and less) and the pH range greater than 8.5. The outcome of this was its introduction at Elizabeth City, N. C., where studies were conducted on a plant scale by L. L. Hedgepeth, in co-operation with the writer. Its use at Chickasaw, Ala., and at other points followed, notably its adoption at Providence, R. I.,[31] and Dallas, Tex. It can be stated with reasonable assurance that ferric iron salts are superior coagulants in dealing with color removal from water and with organic matter removal from other liquids, such as trade wastes and sewages. This statement applies in the matter of costs as well as efficiency.

In some instances, notably when oxygen is present to a sufficient extent and the pH is 9.0, or more, ferrous salts will serve. The chief drawback to the use of unoxidized iron is the fact that effective coagulation is slow, and frequently indifferent. The fact that ferrous hydroxide must first be produced and that thereafter it changes to the insoluble ferric hydroxide, or to some more complex compound, requires more time and longer mixing periods, than is the case where the iron is pre-oxidized, in part if not in whole, before introduction to the liquid being treated. In coagulating creamery wastes ferrous sulfate coagulates primarily as a light and feathery floc of ferrous hydroxide, and but little reaches the ferric state because of lack of oxygen.

Relationship Between pH-Values, Effectiveness of Coagulation, and Unprecipitated (Residual) Iron.—Of particular interest to the writer are the observations of the authors as illustrated in Fig. 10, revealing the pH-zone of quickest floc formation and the most perfect precipitation of the iron added in the form of chlorinated copperas. The curves, indicating that in the zone between pH 4.5 and pH 6.5 iron removal was complete and practically complete between pH 3.8 and pH 7.0, and, similarly, that floc formation was most rapid over the same range, might have been drawn by plotting actual operating data at the filter plant in Elizabeth City, N. C., during the periods of operation with which the writer is familiar.

Using colorless water fortified with calcium chloride the studies made in the laboratory of the University of Iowa show that the floc, if formed at all,

[31] "The Ferric Iron Coagulation of Soft Water," by E. C. Craig and E. L. Bean, *Water Works and Sewerage*, Vol. 79, September, 1932, p. 301.

was very poor between pH 7 and pH 9.5, but the residual iron was satisfactorily eliminated at pH 9.5 and beyond. Such high pH-values can not be considered as of practical use, in dealing with soft organic waters. Color removal would show a minus efficiency because alkalinization not only disperses color, but intensifies it, due to the fact that some of the organic compounds met with in water act as pH indicators to the extent of showing deeper color with increasing pH-values.

In practice, when coagulating with ferric salts at pH less than 7, the appearance of residual iron indicates the need for increased dosage of coagulant; and that is actually done in practice to meet the circumstance rather than reducing the iron applied or increasing the alkali, if the latter is used. It has been demonstrated in laundry waste treatment that acidification with sulfuric acid serves almost the same purpose, provided the necessary minimum of ferric iron is present to produce the floc effectively and give it the high gravity desired.

The authors made some reference to the fact that the quantity of ferric iron was the all-important consideration, regardless of the form in which it is used. In water treatment and sewage coagulation that appears to be approximately correct, but in sewage-sludge conditioning, to the acid side on the pH-scale, it does not hold. Ferric chloride remains superior to chlorinated copperas which probably has 66% of the iron present in the sulfate form, whereas only 33% is converted to the chloride during oxidation of the copperas with chlorine.

New Sources of Ferric Iron.—During the past year there have appeared on the market two ferric sulfate coagulants—one of which is being successfully used at Providence, R. I.[32] Both these products are dry and subject to handling and feeding in the same manner as pulverized sulfate of alumina—filter alum.

[32] "Advances in Iron Coagulation and Coagulants," by E. L. Bean, *Journal*, New England Water Works Assoc., Vol. 47, September, 1933, p. 273.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION MEASURES

Discussion

BY MESSRS. H. H. CHAPMAN, AND E. B. DEBLER

H. H. CHAPMAN,[11] ESQ. (by letter)[11a].—That changes in the water-shed cover upset the state of equilibrium or balance expressed in a certain type and density of vegetation, and that the one effect which may not be immediately apparent, but may be the most far-reaching in the long run is accelerated erosion, as stated by Mr. Sonderegger, is a fact worthy of closest scrutiny in its far-reaching effect on irrigation, storage reservoirs, and water control. This balance is maintained over huge areas in the West by grasses and herbage. Profound disturbance of this equilibrium has taken place through excessive overgrazing. Extensive channel cutting has accompanied this phenomenon in the Southwest and intermountain region. That overgrazing is the principal cause of this arroyo formation is proved by the fact, attested by Dr. Earl Morris, Division of Historical Research, Carnegie Institute, Washington, D. C., that the cycle of degradation now operating is of recent origin. Throughout the occupation of the San Juan Area, in Colorado, by agricultural aborigines, the valley floors were rising instead of being cut away. The recent cuttings have laid bare stratified exposures recapitulating in reverse order the entire culture series as it is known to have existed in the area in question. Other outstanding areas showing the same series are Chaco Canyon and Canyon de Chelly. The destruction of all vegetation by grazing animals, the consequent prevention of reseeding, and the beating out of the roots of grass and other plants, are given as the dominant cause of arroyo cutting in this region by this authority on aboriginal American history.

An exceptionally valuable piece of historical evidence as to the condition of the Southwestern rivers in the period preceding the coming of white civilization is found in the diary[12] of the noted explorer, James M. Pattie.

NOTE.—The paper by A. L. Sonderegger, M. Am. Soc. C. E., was published in December, 1933, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[11] Prof. of Forest Management, School of Forestry, Yale Univ., New Haven, Conn.

[11a] Received by the Secretary January 5, 1934.

[12] "Early Western Travels," by Reuben Gold Thwaites, Vol. XVIII, pp. 87, 131-132.

Of the Gila River in Arizona, now transformed into a typical sandy wash, he says, on December 14, 1824, "we started early and crossed the river, here a beautiful clear stream about 30 yards in width, running over a rocky bottom and filled with fish"; and "on the morning of February 1, 1826, we began to ascend the Black River. It is a most beautiful stream to the point where it forks in the mountains, that is to say, about 80 miles from its mouth." During January, Pattie trapped the Gila to its junction with the Red, or Colorado River, and on February 2, 1826, says,

"At twelve, we started up the Red River, which is between 200 and 300 yards wide, a deep bold stream, and the water at this point is entirely clear. The bottoms are a mile in general width. The timber of the bottoms is very heavy and the grass rank and high."

On April 15, 1826, after passing the Grand Canyon, he says, "we returned to the banks of the Red River, which is here a clear beautiful stream."

The cumulative effects of erosion following disturbance of vegetative equilibrium have been discussed[13] in great detail. B. P. Fleming, M. Am. Soc. C. E., has discussed the problem of water erosion in New Mexico, on the Rio Grande area covered by Mr. Sonderegger's paper, and attributes disastrous changes in stream conditions and erosion to overgrazing.[14]

There are incalculable quantities of loose or unconsolidated soils on all water-sheds, capable of being eroded at an accelerated pace once the vegetative balance is destroyed, and of upsetting the calculations of engineers on storage capacity and life of reservoirs.

E. B. DEBLER,[15] M. AM. SOC. C. E. (by letter)[15a].—The author has devoted not a little space to forestation and rightly so. It is noted that the Forest Service is stated to be engaged upon a program of research with a view, among other objects, of increasing the water yield. Such research, if conducted without prejudice and with due regard to all interests involved, should receive every possible encouragement. Dogmatic propaganda for tree planting without regard to results other than the growing of trees will injure the cause of reforestation. The effects on grazing, recreation, and water production must be given full consideration.

Silt problems arising from accentuated erosion and from stream depletion merit careful consideration by every one. The mining of soils and their wholesale erosion with or without the aid of Man should not be viewed complacently. As with forestation there must be careful consideration of the attendant risks. Silt barriers are being erected or promoted, wholesale. Throughout the United States, many of these erosion checks or barriers are being built of short-lived materials. Who will maintain these barriers? Are they not likely to lead to even greater damage than now occurring when some future flood sweeps through a series of such structures weakened by age and

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[13] Sen. Doc. No. 12, 73d Cong., pp. 298-462 (Copeland Rept.).

[14] "The Southwestern Conservation League," *Bulletin No. 1*, Conservation Ser., Univ. of New Mexico, December 1, 1933.

[15] Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[15a] Received by the Secretary February 19, 1934.

overwhelms an area built up on a false sense of security? Permanent construction and watchful maintenance are essential to the success of such structures with the latter particularly difficult to secure. The efforts of the author and others to arouse interest in the erosion problem are no longer falling on deaf ears; the public is awakening to this menace, but more remains to be done before the selfish interests of the cattle owner, the sheep herder, the agriculturist, and the miner give way to an enlightened public policy.

The writer cannot fully agree with the conclusions presented in the case of the Rio Grande Project. From the Elephant Butte Dam almost to El Paso, Tex., the author's data show increases in river sections, except only where the sections were formerly much greater than the average. The increase in area is by means of bottom scour, occurring even in some of the sections, which show a net loss. The only area that did not show such a scour was one with a total area fully three times the average, and so unnatural in size that the adjustment of river sections and gradients naturally resulted in a reduction under conditions of regulated flow. Section changes have resulted in increased capacities.

The statement that arroyo fans have formed rapids, is also open to question. Bottom scour with the usual increase in section and the lowering of the stream bed at the dams, supports a conclusion that the rapids are the remains of older fans of coarse materials which have resisted recent removal, whereas the softer bottom material between these rapids has been removed. The writer is unaware of data showing the relative silt deposition in the valley above El Paso before and after the construction of the Elephant Butte Reservoir. It is likely that less silt is being carried away from the project than formerly, but he doubts whether the difference is equal to the amount being deposited in the reservoir.

At the International Dam, near El Paso, a silt plug is in constant process of formation through the desilting of diverted waters, little water normally passing this point. Each flood carries away a part of the silt plug. For lack of such floods for a number of years, this plug reached menacing levels prior to 1925 when two floods of local origin restored favorable conditions. The favorable river-bed level of 1907, mentioned by the author, was probably due to severe scouring resulting from the heavy run-off then current, and particularly the high floods of the latter part of 1905.

The stabilized river channel has gradually approached El Paso and should reach there in the near future, with anticipated improvement in local conditions. Thereafter, local floods and surplus waters entering Elephant Butte Reservoir, which have been missing in recent years of sub-normal run-off, no doubt, will clear the river channel at El Paso with sufficient frequency to avoid the extreme high levels of 1925.

Below El Paso, there has been a loss of capacity such as is to be expected in a channel carrying little water for a number of years. This situation will shortly be improved by channel rectification. Unavoidable waste and surplus waters should have little difficulty in maintaining the new channel free of silt accretions provided meandering is prevented.

A feature of stream development not mentioned by the author, and which may warrant much consideration, is the concentration of salts through re-diversion. Rarely is the quality of waters improved in passing through the soils. There is usually a marked increase in deleterious salts, with drain waters in some cases carrying several times the agricultural tolerance limit for ordinary crops. This feature must receive increasing attention as the utilization of Western streams progresses toward complete consumption of all ordinary flows.

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DISCUSSIONS

LOSS OF HEAD IN ACTIVATED SLUDGE AERATION CHANNELS

Discussion

BY H. L. THACKWELL, M. AM. SOC. C. E.

H. L. THACKWELL,[2] ASSOC. M. AM. SOC. C. E. (by letter)[2a].—It is important to select the correct coefficient of n in Kutter's formula, when solving for velocities in open-channel flow, with the conditions of the regimen varying from tabulated values, as registered and advocated by various experimenters and observers. This fact is emphasized in Mr. Townsend's paper.

Present field practice in hydraulics is stressing non-uniform and varied flow in open channels, rather than the rudimentary notions of uniform movement. Striking cases of turbulent flow occur in sewage disposal works. Every form of non-uniform movement may be observed in modern sewage disposal plants, namely, the hydraulic jump as used for mixing chemicals and for aerating purposes; spiral movements in certain types of aerating tanks; wave motion in distributing launders for filters; baffled flow in settling tanks; whirls and eddies obtained in various ways by agitating mechanisms; and diversion of flow directions in conduits, diversion boxes, and over-step aerators.

For some time the writer has felt that the hydraulics of sewage works has been neglected, and that more attention has been given to process treatment than to the mechanics and hydraulics of the subject. The Ganguillet and Kutter formula, for determining the value of c in the older Chezy formula, still has a common acceptance and specific meaning to the average engineer trained in America. With this formula, if the correct coefficient of roughness, n, is chosen for the channel, it is possible to obtain the coefficient, c, in Chezy's formula, and if the measurements of cross-section area, slope, and wetted perimeter are arrived at accurately, an average value of v may be obtained. It is doubtful whether slope determinations influence, or are necessary in obtaining, a correct value of n in the Kutter formula, which was primarily intended for uniform flow conditions.

NOTE.—The paper by Darwin Wadsworth Townsend, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] Cons. Engr., Tyler, Tex.

[2a] Received by the Secretary February 2, 1934.

The author has made observations on the velocities of an open channel having a very turbulent flow, due to the presence of rising air bubbles (resulting in a consequent swirling movement, both vertically and horizontally, and in a positive and negative direction), and he has computed a corresponding value of the coefficient, *n*, to fit such conditions. These observations range from 0.9 ft to 1.3 ft per sec. and, when such velocities are plotted with the coefficient, *n*, Mr. Townsend presents a curve which shows, within these limits, that the value of *n* apparently increases with lowering velocities.

Since the observed depths and varying cross-sectional areas are not given, it is presumed that the full wetted cross-section of the channel was used to obtain the mean velocity in each case, rather than the net cross-sectional area which might have been obtained by subtracting some bottom layer of air and water, where the jet action of the stream of air bubbles presents a definite barrier to forward horizontal flow. The trend of the curve thus plotted was ascertained and extended to zero-velocity limits, where the coefficient, *n*, was 0.23.

With the Chezy formula, as used in this case, the resistance to motion is proportional to the second power of the mean velocity, which is undoubtedly true for all velocities above the critical velocity. At this low critical point, however, the resistance may be proportional to the first power of the velocity.[3] The natural inference is that it would be fictitious accuracy to extend a trend curve beyond the limits of its governing equation. If the percentage of cross-sectional area, uninfluenced by jet action, had been used as a net cross-section, there is reason to believe that the coefficient, *n*, would have increased with a decreasing velocity, but would have been nearly the same for all values of *v*. The difficulty would be to determine this net area uninfluenced by jet action, and yet supporting floating bubbles of air. An air-bubble surface (that is, freely floating and moving with the current of water), would not enter into the calculation of the wetted perimeter, any more than a log floating with a stream current would be considered in obtaining the wetted perimeter of a channel. If the bubble of air were fixed in position, the wetted circumference of such a bubble should be calculated and added to the wetted perimeter of the channel itself.

Before discussing the author's "cause and effect" theory in regard to the varying roughness coefficient, it will be necessary to obtain a better knowledge of the mechanics of the air bubble itself.

In June, 1932, the writer became interested in the subject of the mechanics of the air bubble and the laws governing it, and after searching literature on the subject, was surprised to find that very little experimental data had been published in the past on the subject of air induction into sewage or water. Mr. H. S. Allen made experiments on "The Motion of a Sphere in a Viscous Fluid," and published[4] his findings in 1900. That part of the subject matter dealing with air bubbles was devoted to studying the velocity of very small bubbles rising in water. These bubbles were of such a small size (all less than $\frac{75}{100}$ mm), that the experiments and resulting data were not comparable to the

[3] "Hydraulics and Its Applications," by A. H. Gibson, p. 295.

[4] *Philosophical Magazine*, September, 1900.

larger air bubbles used for agitating water and sewage. Accordingly, the writer decided to make some simple experiments to obtain some definite first-hand information as to the velocity of the bubble rising in water, the size and shape of such bubbles, and their path of travel in ascending to the surface. The apparatus that was selected for the tests consisted of a 5-ft glass cylinder with an inside diameter of 2 in., and having one end swaged and the other open. Both ends were fitted with rubber corks, the one at the swaged end had a single opening for a capillary tube insert, and the cylindrical end had two small tubes inserted in the stopper.

In one of these openings a small glass tube, 5 ft long, was inserted, having the submerged end fused down to a small J-shaped jet of very small bore.

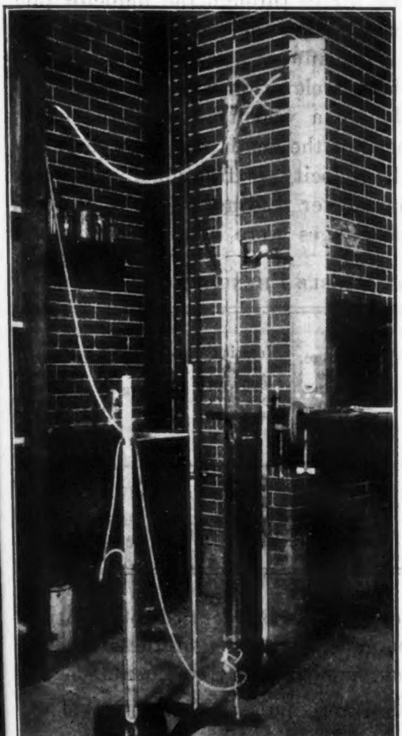

FIG. 6.—APPARATUS SHOWING RISING AIR BUBBLES.

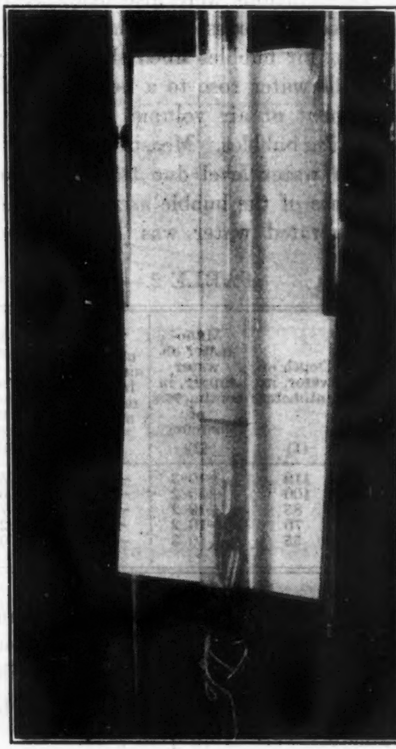

FIG. 7.—AIR-JET MOUTHPIECE SHOWING ACTION AGAINST WATER.

This tube was arranged so that it could be supported at any depth of submergence within the cylinder, thus acting as an air inductor into the sewage. The cylinder was supported vertically, by means of clamps, to the edge of a table. The other opening at the top of the stopper was connected by rubber tubing to a glass Y-tube, one arm of the Y-branch being connected by rubber hose to a mercury manometer, and the other to a vacuum suction pump. The

lower end of the 2-in. cylinder was connected by tube and hose to another mercury manometer (see Fig. 6). The 2-in. cylinder was nearly filled with water. The J-inductor tube was next set in position, so that the jet opening was 4 ft below the surface of the water. The mercury gauges were set in position, and the lower gauge read 10.2 cm of mercury. The vacuum pump was turned on until the suction pressure occurring above the surface of the water was 10.2 cm of mercury; air was then drawn into the water immediately from the atmosphere, and liberated in the form of a stream of bubbles into the water column. Observations were made on the size of the bubbles, their shape and behavior of travel in rising to the surface, and the velocity of ascent.

Observations were also made by forcing air in through the inductor pipe by direct compression, rather than by suction, as by the vacuum process. As soon as air bubbles filled the cylinder in a continuous stream, the surface of the still water rose to a new level; this higher elevation was due to the displacement of air volume, and slightly due to a velocity head, built up by the rising bubbles. Measurements were made of the depth of submergence, the rise in water level due to air volume, the velocity at fractional and whole distances of the bubble ascent, and the manometer readings. The total depth of unaerated water was 141 cm and the gauges were kept at a constant

TABLE 2.—OBSERVATIONS ON BUBBLE PHENOMENA

Item No.	Depth of water, in centimeters (1)	Manometer on water supply, in centimeters of mercury (2)	Manometer on air supply, in centimeters of mercury (3)	Rise of water level due to air volume, in centimeters (4)	Percentage, volume of air in column (5)	Velocity of bubble ascent, in feet per second (6)	Percentage of submergence (7)	Power product, Column (5) by Column (7) (8)
1	119	+10.3	—10.2	7.5	5.92	0.78	84.5	5.00
2	100	+10.2	—10.2	9.0	8.25	1.09	71.0	5.86
3	85	+10.2	—10.2	9.7	10.25	1.31	60.2	6.16
4	70	+10.2	—10.2	9.2	11.6	1.45	49.8	5.79
5	55	+10.3	—10.3	8.0±	12.6	1.68	39.02	4.93

pressure of 10.2 cm of mercury. A number of series of identical observations was made with slight variations of submergence. One set of these observations is given in Table 2 with the following supplementary comments:

Item No. 1.—At this stage the bubbles were pulsating slightly. Their diameters were from $\frac{1}{4}$ to $\frac{1}{2}$ in.

Item No. 2.—This was the beginning of marked turbulent flow, with bubbles coalescing.

Item No. 3.—Turbulence increased at this point so that large bubbles dominated the smaller ones.

Item No. 4.—Turbulence still further increased, so that small bubbles were held down by the larger ones.

Item No. 5.—Violently turbulent flow, such that the operator was unable to measure the velocity except by extending the velocity curves.

From the data collected in several series, curves were constructed, as shown in Fig. 8, and from these curves the following formulas are derived (approximately straight-line variations on the various series of data):

$$Y = 0.15\ (100 - X) + 4 \dots\dots\dots\dots\dots\dots\dots (1)$$

and,

$$V = 0.0192\ (100 - X) + 0.5 \dots\dots\dots\dots\dots\dots\dots (2)$$

in which, X = percentage of submergence (expressed as whole number); Y = percentage of air in a column of rising air; and V = vertical velocity, in feet per second, of rising bubbles.

By plotting the power product (Column (8), Table 2) against submergence (Column (7), Table 2) a maximum of aeration is indicated at 60% submergence (see Fig. 8, Curve C), with the provision that the air pressure is

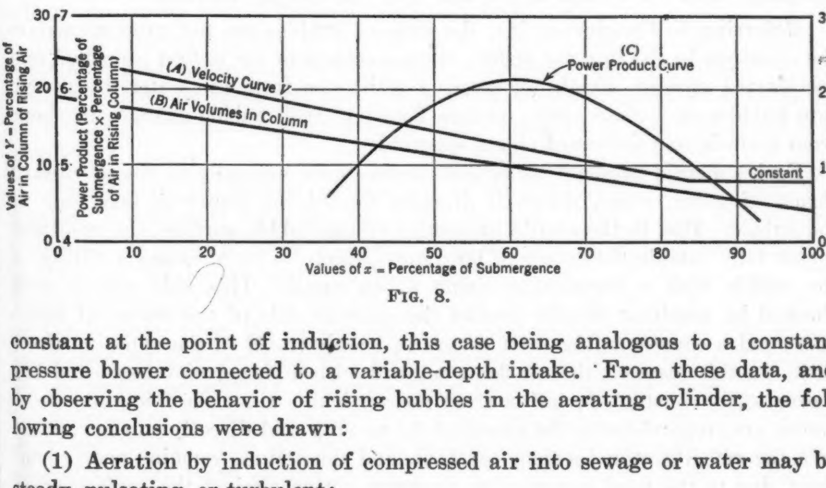

FIG. 8.

constant at the point of induction, this case being analogous to a constant pressure blower connected to a variable-depth intake. From these data, and by observing the behavior of rising bubbles in the aerating cylinder, the following conclusions were drawn:

(1) Aeration by induction of compressed air into sewage or water may be steady, pulsating, or turbulent;

(2) With steady or pulsating conditions, the percentage of air in the cross-section is less than 7 by volume; and with all bubbles larger than a critical size (about 1 mm), the upward vertical velocity is constant and approximates 0.8 ft per sec (furthermore, this velocity is non-accelerating after the air has started to float freely as an individual bubble);

(3) With turbulent flow the upward velocity is non-accelerating, and varies in almost direct proportion to the percentage of air in the rising column (this velocity may vary from 0.8 ft per sec to 2 ft per sec, which upper limit is reached when the air volume of displacement is 16% of the water volume and at 22% submergence, provided the air pressure is constant at the point of induction);

(4) Bubbles have the same velocity in rising through water, whether they have been introduced by means of positive pressure, or by negative pressure (this deduction, of course, agrees with theory);

(5) The percentage of air injected is in direct proportion to the percentage of submergence, when the air is inducted at a constant pressure, and with the same type of injector orifice;

(6) The maximum inducted air remaining in the water at any one time, occurs when there is a submergence of 50%, and at this depth the aeration appears to be the most efficient (see Curve *A*, Fig. 8);

(7) The specific gravity of air mixed with water is in direct proportion to the volume of admixture; but when bubbles begin to appear and confined water is displaced upward, the specific gravity may be taken as that of water, provided no allowance has been made for the added depth of water column (temperature and atmospheric pressure not being considered);

(8) Air bubbles have a certain critical size in flotation, at which volume the shape changes from circular to lense or oblate spheroid; and,

(9) The heights of influence of jet action varies directly with the submergence, with the volume of air, and with the diffuser medium.

Referring to Conclusion (8), the sizes of bubbles are not proportional to the openings in the injector orifice, since a stream of air is first propelled and accelerated upward, which, on meeting resistance from the water, breaks up into bubbles of various sizes; on free flotation, each bubble changes in shape from a circle to a deformed oblate spheroid.

Each spheroid presents its largest surface area normally to the downward thrust of water, which thrust is directed toward the center of buoyancy of the bubble. Due to the elastic properties of the bubble surface, the resultant thrust falls outside the center of buoyancy, which, in turn, causes a tilting of the bubble with a consequent rising at an angle. This side slip is soon checked by resultant thrusts toward the opposite side of the center of buoyancy, and a corresponding shift occurs in bubble travel in the opposite direction. In this manner, the air floats upward with a zigzag movement and has a practically constant deviation from the vertical. Since the frictional resistances are proportional to the square of the velocity and directly to the exposed area the velocity soon becomes constant, and the volume remains nearly constant, due to the total compressive pressures acting against the surface area; these pressures are a summation of hydrostatic pressure and of velocity friction head. Apparently, these forces act automatically, and the size and velocity of the bubble remain practically the same throughout its travel range. Coalescence is the main cause of increased bubble size, but this factor will not cause increased velocities, or accelerating velocities, provided the proportion of entrapped air to water volume remains the same.

Referring to Conclusion (9), the minimum distance above the orifice, before the air bubble is freely floating and outside the influence of jet velocity, is about 6 in., and the maximum distance likely to occur in any ordinary orifice with various volumes and free flotation is 18 in.

The writer is aware of the fact that the foregoing observations are not conclusive, because the experiments were not sufficiently extensive and did not include the factors of temperature, humidity of air within the bubble of air itself, the atmospheric pressure, and the effect of interference within a cylinder of small caliber (2 in.). There was also a certain lack of precision

and refinement in the measurements, the velocity being timed by stop-watch. However, the observations gave a fair idea of what was actually occurring and cleared up some points which, at first, seemed paradoxical; that is, that the bubbles do not have an accelerated velocity, and that the bubble does not increase in volume on rising through a lowered water pressure.

Returning now to the original discussion of the flow of sewage through an aerating launder, it is proper to conclude that the resistance to flow is caused by the following factors:

(*a*) The natural coefficient of friction of the sides and bottom of the channel itself.

(*b*) The resistance offered, mainly near the bottom of the channel, by the jet action or barrage of air particles which are traveling upward and floating horizontally with the current;

(*c*) The resistance due to cross-flow, or displacement flow under each rising bubble (this resistance will increase in direct proportion to the percentage of volume of air within the cross-section); and,

(*d*) The resistance due to vertical flow, upward and downward currents, which occur because unbalanced pressure columns have a higher or lower density caused by contained air. (The actual distance traveled by a particle of sewage is several times greater than the measured horizontal distance of flow.)

Referring to Resistance (*c*), the path of the bubbles, traveling upward and floating sidewise, is measured by vector analysis. The higher the channel velocity, the more bubbles will remain in the sewage at any one time, on account of the longer travel distance before bursting at the surface. The lower the channel velocity, the greater is the resistance offered by the barrage at the bottom, since the air-jet has less deflection and such jet influence is continued for a longer time interval; however, there will be fewer air bubbles in the sewage on account of shorter travel in both time and distance.

A resultant of Resistances (*a*) to (*d*) will certainly slow down the velocity very appreciably. The fact remains, however, that the cross-section has been diminished far more by the influence of the bottom jet action in the case of low-velocity flow (having a constant volume of air per square foot of bottom area) than in the case of high-velocity flow. The agitation in the case of low flow should be greater than in the case of high flow, due to the increase of air to sewage ratio.

It would seem fairer to allow a deduction in area on account of jet action, rather than to show that a low velocity increases the coefficient of roughness, n, in Kutter's formula which, on the face of it, is contrary to theory.

In conclusion, the writer is grateful for this interesting paper by Mr. Townsend and for the light it has thrown on the subject of the hydraulics of aeration channels which, heretofore, has been much neglected.

It is to be regretted that the full data of the experiments were not published, such as the varying wetted cross-sections at different rates of measured flow. More experiments on this subject will be required before an accurate forecasting of the roughness coefficient, n, may be made on sewage aeration channels.

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DISCUSSIONS

AN APPROACH TO DETERMINATE STREAM FLOW

Discussion

BY C. S. JARVIS, M. AM. SOC. C. E.

C. S. JARVIS,[10] M. AM. SOC. C. E. (by letter)[10a].—This paper embodies several forward steps in a scientific field of much promise. The author has made practical application of some of the most recent developments toward logical and fairly dependable estimates of stream flow that should follow rainfall of given intensities and duration.

The assignment of proper and consistent values for the various constants seems to demand special skill or experience for guidance. The values of V in Table 2, ranging from 0.0000195 to 0.0001570 (or to eight times the smaller), need some kind of fundamental co-relation. Obviously, the factors in Table 2 most directly related are the length, L, and some function of the slope of the main channel, designated as S. Furthermore, they vary inversely, as shown by Equation (1). By inspection and trial, it was found that a fair approximation to the tabulated values of V for the six streams, as there listed, may be computed from the following expression,

$$20 \frac{\sqrt{S}}{L} = V \text{ (approx.)} \qquad (5)$$

Slide-rule computations brought the results shown in Table 5. While the agreement is not all that could be desired, it may served as a first approximation.

Examination of the extensive tabulations leading up to the desired hydrograph, as illustrated in Fig. 3, suggests the idea that a free-hand sketching of a curve under the pluviagraph (with ordinates varying from 1% to 30% of the corresponding pluvigraph values, and, with due regard to the time element for the given water-shed, the mounting percentage of run-off with higher degrees of saturation, after several days of storm, and whatever local data are available), may result in nearly as close approximations of the observed hydro-

NOTE.—The paper by Merrill M. Bernard, M. Am. Soc. C. E., was published in January, 1934, *Proceeding*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[10] Prin. Hydr. Engr., U. S. Engr. Office, St. Paul, Minn.

[10a] Received by Secretary February 1, 1934.

graph as will normally follow from the more elaborate and detailed procedure. It is conceivable that the selection of the numerous constant values, the mastery of the extensive nomenclature, and the great number of computations involved in the author's method of determining stream flow, may be too much to expect of those who attempt to follow it. The value of dependable forecasts for different types of storms that habitually visit a region cannot be over-estimated; but it will require specialists in the science of hydrology and in the application of both meteorological and hydraulic data, to prepare such charts, with coefficients of variation to conform with seasonal and other changes of physical data affecting run-off.

TABLE 5.—VALUES OF V IN RELATION TO S AND L IN TABLE 2, IN MILLIONTHS.

River	From Table 2	Computed by Equation (32)	River	From Table 2	Computed by Equation (32)
Raccoon Creek............	61.0	49.6	Walhonding River........	42.0	33.7
Licking River............	157.0	128.2	Tuscarawas River........	32.0	33.2
Hocking River............	54.0	50.8	Muskingum River.........	19.5	22.4

Before 1864 it was known that the annual run-off from the Mississippi water-shed was approximately "one-fourth of the downfall," or precipitation. Subsequent years of record, with gauging and meteorological stations multiplied in number, have demonstrated that the yield may range between approximately 14 and 31% of the rainfall; but these later records confirm the early investigations as to average percentage of yield. Therefore, a pluviagraph of this area would constitute a fairly reliable guide for a resulting hydrograph, with its peak covering the months of April and May, and its area between the curve and the horizontal axis representing about 25% of the area under the pluviagraph. For such a water-shed as that of the Mississippi, the human or personal equation would be kept within bounds more effectively by this means than if independent judgments were required to assign proper values to various constants in the several equations utilized, especially as some of these values constitute powers of certain factors. Furthermore, due regard to the grouping of rainfall days, and to the seasons in which they occurred, would justify a departure from the average value of 25% as representing run-off within the observed range of 14 to 31%; and, in this manner, the sketched hydrograph may prove to be as reliable as those resulting from voluminous computations, interspersed with occasional estimates that are just as difficult to apply and as far-reaching in their effects as those required in sketching the desired hydrograph from the pluviagraph.

After investigating or dealing with the entire category of reputable run-off formulas now available, the writer continues to incline toward simplification rather than toward multiplication of factors and processes. For first approximations to maximum, or for any other assumed rate of run-off corresponding to the probable frequency of recurrence, applicable to any given section of the United States, or of other countries, significant basic data have been

presented by the writer.[11] The formula for maximum run-off which he has recommended is,

$$Q = 10\,000\; p \sqrt{M} \qquad (6)$$

in which, Q is the peak discharge, in cubic feet per second; M, the drainage area, in square miles, and p, the percentage rating on the Myers scale, roughly equal, numerically, to the percentage of rainfall that appears as run-off during the most severe storms and floods of that area, except as modified by geometrical shape of basin and alignment of main drainage channels, or other water-shed characteristics. The streams of Northern Ohio are found to range from about 5 to 12% on the Myers scale, while those of Southern Ohio are usually 20% or more. This disparity is traceable partly to the difference in rainfall habits, but, in greater degree, to the difference in soil and surface characteristics, channel gradients, and valley storage. The maximum discharges recorded in Table 1 are found to rate, respectively, only 1.4%; 1.3%; 2.3%; 2.9%; 1.7%; and 6.8% on the Myers scale. The last-named item applies to the Muskingum River, which attained a rating of 28.4% at Marietta, Ohio, in 1913. The foregoing percentages, therefore, represent run-offs of relatively low volume and correspondingly high frequency of recurrence, perhaps attainable nearly every year.

[11] "'Flood Flow Characteristics," *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 985; and "Rainfall Characteristics and Their Relation to Soils and Run-Off," *Loc cit.*, Vol. 95 (1931), p. 379.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ON THE BEHAVIOR OF SIPHONS

Discussion

By Herbert H. Wheaton, Assoc. M. Am. Soc. C. E.

Herbert H. Wheaton,[10] Assoc. M. Am. Soc. C. E. (by letter)[10a].—In the "Conclusion" to his paper Mr. Stevens observes that "some of the phenomena and behavior of the siphons herein outlined could never be detected in the laboratory by the use of models." The writer feels that he should add a few remarks about the tests he made of models of Siphon No. 7 (see Fig. 3(*c*)) of the Leaburg siphons, at the hydraulic laboratory at the University of California, shortly after he assisted Mr. Stevens in making the field tests.

Models of Siphon No. 7 to scales of 1:20, 1:15, and 1:7.5 were tested. The 1:20 model was fitted with piezometer tubes corresponding to those in the prototype. The pressure readings at Sections 1 and 2, intake and throat (see Fig. 7), checked those in the full scale almost exactly. At the other sections this was not the case, because the model ran full.

The coefficients of discharge of the models, based upon the formula, $Q = CA \sqrt{2gH}$, were 0.68 for the 1:20; 0.69 for the 1:15; and 0.71 for the 1:7.5 scales. The coefficient corresponding to these for the original was 0.63. Had the full-scale siphon run full it would have shown a coefficient of discharge of something in excess of 0.71, the value of which could be found by extrapolation from the model results. The model tests demonstrate the maximum coefficient for a siphon of this design. The scale effect is due to the fact that the discharge formula is based upon the assumption that only gravitational forces influence the discharge. The coefficients given for the models are constant for velocities in the models which give a value of Reynolds' number greater than some fixed value which is the same for all scales of model[11].

The writer thought that Siphon No. 7 of the Leaburg siphons probably leaked enough air around the movable air-vent cover-plate to have some effect

Note.—The paper by J. C. Stevens, M. Am. Soc. C. E., was published in August, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1933, by I. M. Nelidov, Assoc. M. Am. Soc. C. E.

[10] Fresno, Calif.

[10a] Received by the Secretary December 20, 1933.

[11] "Experiments on Siphon Spillways," by A. H. Gibson, T. H. Aspey, and F. Tattersall, *Minutes of Proceedings*, Inst. C. E., Vol. 231, 1931.

upon the discharge. He made tests, therefore, to determine the effect of air leakage upon the 1:20 and 1:15 scale models. He found that the coefficient of discharge for each of these models could be reduced to 0.63 if the model were allowed to suck from 3 to $4\frac{1}{2}\%$ of air at the air vent; that is, if the volume of air measured at atmospheric pressure were 3 to $4\frac{1}{2}\%$ of the volume of water passing through the siphon. Although no doubt some air leakage occurred at the intake of the prototype, the writer is not prepared to say whether it could have been in such quantities.

Mr. Stevens mentions that Siphon No. 7 primed with a head of water over the crest of 0.2 to 0.3 ft. The three models, when air-tight and sucking no air at the intake, checked each other almost exactly as to the head necessary to prime them for corresponding rates of rise of the forebay water level, but all showed priming heads far in excess, correspondingly, of those in the original. When the water rose in the forebay it could not rise to the same level inside the siphon throat because of air compression within the tube. The quantity of water flowing over the crest was not enough to produce a velocity in the sealing basin sufficient to carry the air out to the outlet until a much higher level, correspondingly, than 0.3 ft above the crest was reached in the forebay.

The writer concluded that the air pressure within the tube of the prototype never became greater than atmospheric, or at the most very little greater, perhaps because some air could escape around the air vent, and also because of the larger volume of the tube. He, therefore, connected the throat of the 1:7.5 model with the outside air by a $\frac{3}{16}$-in. rubber hose so that compressed air could escape during priming. Observation of the water levels both in the forebay and within the siphon showed that they remained the same. The model then primed at heads that corresponded almost exactly with a head of 0.3 ft in the full scale. It was interesting to note that the tube could be left in the throat all during the priming, permitting air to enter the throat when pressure there became less than atmospheric, and that this made no noticeable difference in the head necessary for priming. After the velocity in the sealing basin became sufficient to carry the air bubbles around the lower bend, the tube was evacuated so rapidly that the small hose opening made no appreciable difference.

APPLICATIONS FOR ADMISSION AND FOR TRANSFER

The Constitution provides that the Board of Direction shall elect or reject all applicants for *Admission* or for *Transfer,* and, in order to determine justly the eligibility of each candidate, the Board must depend largely upon the Membership for information.

This list is issued to members in every grade for the purpose of securing all such available information, and every member is urged to scan carefully each monthly list of candidates and to furnish the Board with data in regard to any applicant which may aid in determining his eligibility. It is the *Duty* of all *Members* to the *Profession* to assist the *Board* in this manner.

It is especially urged, in communications concerning applicants, that a *Definite Recommendation as to the Proper Grading in Each Case* be given, inasmuch as the grading must be based upon the opinions of those who know the applicant personally as well as upon the nature and extent of his professional experience. If facts exist derogatory to the personal character or to the professional reputation of an applicant, they should be promptly communicated to the Board. *Communications Relating to Applicants are considered by the Board as Strictly Confidential.*

The Board of Direction will not consider the applications herein contained from residents of North America until the expiration of thirty (30) days, and from non-residents of North America until the expiration of ninety (90) days from March 15, 1934.

MINIMUM REQUIREMENTS FOR ADMISSION

Grade	General Requirement	Age	Length of Active Practice	Responsible charge of work
Member	Qualified to design as well as to direct important work	35 years	12 years*	5 years of important work
Associate Member	Qualified to direct work	27 years	8 years*	1 year
Junior	Qualified for sub-professional work	20 years†	4 years*	
Affiliate	Qualified by scientific acquirements or practical experience to co-operate with engineers	35 years	12 years*	5 years of important work
Fellow	Contributor to the permanent funds of the Society			

*Graduation from a school of engineering of recognized reputation is equivalent to 4 years of active practice.

†Membership ceases at age of 33 unless transferred to higher grade.

The fact that applicants give the names of certain members as references does not necessarily mean that such members endorse.

FOR ADMISSION

BAUER, CARL HENRY, Evanston, Ill. (Age 39.) State Engr. Inspector for Illinois, P. W.A. Refers to R. W. Emmert, B. G. Leake, M. Rosner, B. B. Shaw, C. H. Trask.

BEFFA, THEODORE ALBIN, St. Louis, Mo. (Age 35.) Engr. with City of St. Louis. Refers to W. C. E. Becker, L. R. Bowen, W R. Crecelius, E. C. Renard, E. O. Sweetser.

BROWN, GEORGE FRANK, Los Angeles, Cal. (Age 47.) Refers to J. C. Allison, H. T. Cory, A. Jones, G. W. Jones, J. E. Rockhold, A. K. Warren.

BRUMUND, GERRY HENRY, Sacramento, Cal. (Age 25.) Jun. Engr., Topographic Branch, U. S. Geological Survey. Refers to F. L. Bixby, H. P. Boardman, H. H. Hodgeson, C. N. Mortenson.

CALVERT, JOHN THORNTON, London S. E. 24, England. (Age 26.) With John Taylor & Sons, Chartered Civ. Engrs. Refers to T. R. Camp, W. Donaldson, H. P. Eddy, H. P. Eddy, Jr., G. W. Fuller, G. B. Gascoigne.

CLUTE, HAROLD MOORE, Knoxville, Tenn. (Age 33.) Jun. Engr., Tennessee Valley Authority. Refers to A. C. Bux, F. T. Darrow, M. I. Evinger, D. H. McCoskey, J. G. Mason, C. E. Mickey, A. L. Ogle.

CONESA, HAMLET PEDRO, Ponce, Puerto Rico. (Age 30.) Engr. in charge, Dept. of Eng., Central Mercedita. Refers to T. F. Bowe, J. M. Canals, G. W. Fuller, J. M. Giles, A. S. Lucchetti-Otero, A. Nones, F. Pons.

COZZENS, HOWARD FRANCIS, Salinas, Cal. (Age 47.) County Surveyor and Supt. of Highways and Bridges, Monterey County. Refers to H. J. Brunnier, C. C. Cottrell, A. P. Denton, J. R. Fox, C. G. Gillespie, A. J. Grier, C. P. Jensen, R. M. Morton, C. W. Petit, C. S. Pope, A. G. Proctor, E. J. Schneider, C. A. Smith, O. C. Tretten.

CROCKER, FORREST SAMUEL, Castle Rock, Colo. (Age 28.) Rodman, U. S. Geological Survey. Refers to F. R. Dungan, C. L. Eckel, R. Follansbee, R. C. Gowdy, M. S. Ketchum, A. O. Ridgway.

CULLINAN, ROY BERNARD, North Abington, Mass. (Age 24.) With Massachusetts State Highway Eng. Dept. Refers to W. E. Corbett, H. A. Gray, E. Harsch, J. A. Johnston, E. D. Kingman, T. W. Proctor.

DANDA, FRANK ANTHONY, Cicero, Ill. (Age 31.) Engr. in charge of C. W. A. projects for Cicero Board of Education, Dist. No. 99. Refers to N. E. Anderson, L. B. Barker, J. R. Griffith, P. M. Larsen, J. F. Mangold, H. Penn, R. L. Stevens, W. J. Wagner.

FORBESS, ORDIS ELDON, Tulia, Tex. (Age 21.) Rodman, Texas State Highway Dept. Refers to O. V. Adams, W. H. Garrett, J. H. Murdough.

GETCHELL, WENDELL BRUNGART, Uniontown, Pa. (Age 24.) Draftsman and Checker, Pennsylvania Dept. of Highways, Unit 12. Refers to A. Diefendorf, L. C. McCandliss.

HOGAN, FRANK, New York City. (Age 33). Occasional jobs on general engineering work. Refers to D. Gutman, S. E. Page, N. A. Richards, H. V. Spurr, J. Tarnay.

JENSEN, EMMANUEL TRANBERG, Ames, Iowa. (Age 23.) Research Asst., Iowa Eng. Experiment Station. Refers to A. H. Fuller, W. J. Schlick, M. G. Spangler.

JEWETT, GEORGE EDGAR, Murphysboro, Ill. (Age 26.) State Foreman, Camp 66 P. E. Refers to H. Cross, W. C. Huntington, W. M. Wilson.

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KINDSVATER, EMIL FRED, Bartlesville, Okla. (Age 33.) Engr., Phillips Petroleum Co. Refers to E. Boyce, H. W. Crawford, J. O. Jones, W. C. McNown, H. A. Rice, A. H. Riney.

KINGMAN, IRVING HALL, Flushing, N. Y. (Age 22.) Refers to F. A. Barnes, L. G. Holleran, A. B. Miller, J. E. Perry, R. Y. Thatcher, A. U. Whitson.

KOESSNER, ERWIN, Seattle, Wash. (Age 22.) Refers to G. E. Hawthorn, C. C. More.

KONSTANT, NICHOLAS ZACHARIA, Chicago, Ill. (Age 39.) Structural Engr. Refers to L. D. Gayton, J. R. Griffith, I. F. Stern, N. M. Stineman, W. J. Titus.

LAWRENCE, CHARLES, Upper Darby, Pa. (Age 41.) Senior Draftsman, Pennsylvania State Highway Dept. Refers to L. M. Allison, B. L. Green, S. H. McCrory, F. Mauro, H. B. Miller, W. H. Russell, E. E. Sousley.

LOWRY, ROBERT LEE, JR., Austin, Tex. (Age 34.) Tech. Asst., Texas State Reclamation Dept. Refers to E. C. H. Bantel, B. B. Brier, J. S. Broyles, C. S. Clark, A. H. Dunlap, C. E. Ellsworth, S. W. Freese, F. S. French, J. A. Norris, E. N. Noyes, T. U. Taylor, A. M. Vance, R. G. Waggener, G. G. Wickline, B. F. Williams.

McCONNELL, JOHN WALDO, Dobbs Ferry, N. Y. (Age 29.) Field Engr., Westchester County Park Comm. Refers to H. G. Balcom, C. W. Comstock, D. M. Garber, J. C. Hoyt, G. Schobinger, B. S. Thayer.

McKAY, DONALD MIDDLETON, Rochester, N. Y. (Age 28.) Bill of Materials Clerk, Eng. Dept., The Pfaudler Co. Refers to W. E. Belcher, E. L. Eriksen, J. W. Moffett, E. S. Ovenshine, G. Schobinger, R. B. Wiley.

McLEOD, EVAN WAYNE, Carson City, Nev. (Age 28.) Res. Engr., Nevada State Highway Dept. Refers to E. C. Brown, C. L. Hill, T. R. King, G. W. Malone, W. H. Smith.

MELTON, THORNTON CARTER, Culpeper, Va. (Age 34.) Chf. of Survey Party, Virginia State Highway Comm., Richmond, Va. Refers to H. T. Ammerman, A. H. Bell, J. J. Cassaday, B. P. Harrison, L. Mackey, W. W. McClevy.

MULHOLLAND, ANDREW NEWELL, New York City. (Age 29.) Asst. Supt. Columbia Eng. & Contr. Co. Refers to A. Haring, G. G. Honness, E. G. Hooper, T. Merriman, C. T. Schwarze, C. L. Spaulding.

MURDICHIAN (Formerly MUGRDICHIAN), KARMY KARNIG, Brooklyn, N. Y. (Age 26.) Draftsman, Gibbs & Hill, Cons. Engrs. New York City; also (evenings) Instructor, Eng. Div., New York Univ. Refers to A. D. Fields, A. Haring, J. F. Krakauer, C. T. Schwarze, D. S. Trowbridge.

MYERS, MASON KAY, Sacramento, Cal. (Age 22.) Jun. Topographic Engr., U. S. Geological Survey. Refers to F. L. Bixby, H. P. Boardman, H. H. Hodgeson, C. N. Mortenson.

PINEO, CHARLES STANLEY, Balboa, Canal Zone. (Age 29.) Asst. Engr., The Panama Canal. Refers to L. V. Branch, R. M. Conner, A. C. Eaton, R. L. Klotz, R. F. Olds, E. S. Randolph.

RAYMOND, SENIUS JOHN, Fort George G. Meade, Md. (Age 44.) Capt., Q. M. C., U. S. Army. Refers to W. T. Ballard, E. A. Ballou, A. A. Fries, C. C. Key, M. A. Long, A. W. Parker.

READ, LOTAN CHILSON, Sandusky, Mich. (Age 53.) County Surveyor and Cons. Engr., Sanilac County, Mich.; also Cons. Engr., Craig Gold Mine, Ontario, Canada. Refers to A. W. Buel, W. W. DeBerard, G. C. Dillman, C. W. Hubbell, R. H. Merrill, L. J. Rothgery, J. R. Rumsey.

SAMUEL, THOMAS DUNCAN, III, Kansas City, Mo. (Age 27.) Draftsman, Black & Veatch, Cons. Engrs. Refers to J. F. Brown, E. H. Dunmire, W. G. Fowler, A. P. Learned, W. C. McNown, F. A. Russell, F. M. Veatch, N. T. Veatch, Jr.

SPENCER, WALTER EARL, Spirit Lake, Idaho. (Age 29.) Rodman with H. T. Evans. Refers to I. N. Carter, I. C. Crawford, J. W. Howard, R. W. Spencer.

STAUB, WILLIAM SHAFFER, Weston, W. Va. (Age 23.) Weston Water Co. Refers to L. V. Carpenter, R. P. Davis.

SWANTON, WALTER FREDERICK, Denver, Colo. (Age 23.) Jun. Engr., Dam Designing Sec., Bureau of Reclamation, U. S. Dept. of Interior. Refers to O. B. French, J. J. Hammond, J. C. Moses, H. W. Tabor, R. F. Walter.

TAWSE, BERTRAM WILLIAM, Kirkcudbright, Scotland. (Age 27.) Asst. Res. Engr., *Sir* Alexander Gibb and Partners, London, S. W. I., England. Refers to J. Forgie, R. Freeman, A. Gibb. (Applies in accordance with Sec. 1, Art. 1, of the By-Laws.)

TURRENTINE, ROBERT EMMETT, Jr., Houston, Tex. (Age 29.) Contr. Refers to R. J. Cummins, J. B. Dannenbaum, J. C. McVea, L. B. Ryon, Jr., L. H. Schlom.

VALLIN, ALBERT RICHARD, Alpine, Ariz. (Age 26.) With U. S. Bureau of Public Roads, State of Arizona. Refers to H. B. Elmendorf, B. P. Fleming, G. L. McLane.

VAUGHN, ROBERT, Tacoma, Wash. (Age 48.) Road Supervisor, Commr.'s Dist. No. 3, Pierce County, Wash. Refers to C. D. Forsbeck, J. P. Hart, N. F. Jahn, K. C. McFarland, C. E. Putnam, A. M. Truesdell.

WILLIAMS, ALOYSIUS JOHN, Jackson Heights, N. Y. (Age 29.) Eng. Asst. Dept. of Sanitation, New York City. Refers to A. R. Glock, R. H. Gould, N. I. Kass, D. Ramsay, W. L. Sylvester, G. R. B. Symonds, O. Wolpert.

ZUVICH, THOMAS JOSEPH, Brooklyn, N. Y. (Age 25.) Refers to E. L. Clarke, D. D. Curtis, H. E. Glenn.

FOR TRANSFER

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GREEN, JOHN WESLEY, Assoc. M., San Francisco, Cal. (Elected March 11, 1929.) (Age 41.) Associate Bridge Engr., California Div. of Highways, Sacramento, Cal. Refers to C. E. Andrew, H. J. Brunnier, M. C. Collins, S. S. Gorman, F. W. Panhorst, D. R. Warren, G. D. Whittle.

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HUSSEY, HAROLD DUDLEY, Assoc. M., New York City. (Elected Oct. 21, 1924.) (Age 45.) Designing Engr., American Bridge Co. Refers to C. R. Harding, O. E. Hovey, R. Khuen, Jr., E. H. Rockwell, J. E. Wadsworth.

KRAMER, HANS, Assoc. M., Memphis, Tenn. (Elected Junior Nov. 25, 1919; Assoc. M. Dec. 15, 1924.) (Age 39.) Asst. to Dist. Engr. (Area Engr.), U. S. Engr. Office. Refers to F. C. Boggs, L. L. Calvert, T. H. Jackson, G. B. Pillsbury, W. F. Schulz, F. D. Shaw, B. B. Somervell.

FROM THE GRADE OF JUNIOR

ADAMS, THOMAS CALDWELL, Jun., Salt Lake City, Utah. (Elected May 19, 1924.) (Age 32.) Associate Prof. of Civ. Eng., Univ. of Utah; also with U. S. Coast and Geodetic Survey as Special Representative for Utah. Refers to R. K. Brown, R. A. Hart, H. S. Kerr, R. B. Ketchum, G. D. D. Kirkpatrick, A. B. Purton, F. H. Richardson.

CARNEY, JAMES THOMAS, Jr., Jun., Stephenville, Tex. (Elected Aug. 18, 1930). (Age 32.) Cultural Foreman, National Park Service. Refers to H. A. Beckwith, C. S. Clark, F. A. Dale, J. P. Gallager, G. B. Keesee, J. A. Norris, K. K. Prestridge, W. L. Rockwell, P. A. Welty.

ELLIS, MERLE WILSON, Jun., Bishop, Cal. (Elected Nov. 11, 1929.) (Age 30.) Asst. Highway Engr., State Dept. of Public Works, Div. of Highways, Bishop, Cal. Refers to E. L. Driggs, H. B. Fisher, F. W. Hanna, R. C. Kennedy, J. S. Longwell, J. Munn, G. B. Sturgeon, Sr.

GESSNER, EDWARD HEIM, Jun., New Orleans, La. (Elected Oct. 26, 1931.) (Age 27.) Field Engr. with F. Shutts & Sons. Refers to T. N. C. Bruns, J. F. Coleman, R. J. Cummins, D. Derickson, J. P. Ewin, W. B. Gregory, B. H. Grehan, P. V. Pennybacker, A. M. Shaw, E. E. Shutts, L. C. Smith.

HAYES, NATHANIEL PERKINSON, Jun., Greensboro, N. C. (Elected Aug. 29, 1927.) (Age 32.) Sales Mgr., Carolina Steel & Iron Co. Refers to A. F. Armstrong, H. G. Baity, W. R. Davis, D. M. Garber, T. F. Hickerson, R. MacMinn, S. R. Webb.

HENDON, HARRY HOLMAN, Jun., Birmingham, Ala. (Elected Feb. 23, 1932.) (Age 30.) San. Engr., San Dept., Jefferson County. Refers to F. Bachmann, J. A. C. Callan, W. H. Caruthers, G. J. Davis, Jr., A. C. Decker, C. J. Rogers, W. G. Stromquist.

LIVESAY, DURWARD PAUL, Jun., Olympia, Wash. (Elected Oct. 14, 1930.) (Age 32.) Bridge Designer, Washington State Highway Dept. Refers to C. E. Cleaver, F. C. Dunham, O. R. Elwell, R. W. Finke, H. H. Gilbert, M. A. Gould, H. H. Jordan, L. V. Murrow, M. S. Woodin.

MILONE, MICHAEL EDWARD, Jun., New York City. (Elected June 4, 1928.) (Age 32.) Asst. Engr., Board of Transportation, 2d Div. Refers to H. M. Bergman, R. E. Goodwin, H. W. Lesh, F. O. X. McLoughlin, J. C. Rathbun, P. Sander.

PAJOT, CLAYTON JAMES JOSEPH, Jun., Detroit, Mich. (Elected Nov. 28, 1932.) (Age 32.) Instructor, Eng. Mechanics, Univ. of Detroit. Refers to E. L. Eriksen, P. A. Fellows, J. T. N. Hoyt, F. H. Nygren, J. A. Van den Broek.

TAPLEY, GEORGE MANNING, Jun., Clarendon, Va. (Elected Dec. 14, 1925.) (Age 31.) Civ. Engr. (San.) with Quartermaster General, U. S. Army, Washington, D. C. Refers to A. L. Anderson, W. B. Carr, G. R. Clemens, C. B. Hawley, W. H. McAlpine, R. F. Rhodes, F. G. Switzer.

TIMBY, ELMER KNOWLES, Jun., Princeton, N. J. (Elected June 10, 1929.) (Age 28.) Instructor in Eng., School of Eng., Princeton Univ. Refers to O. H. Ammann, C. E. Andrew, A. H. Baker, G. E. Beggs, F. H. Constant, R. E. Davis, L. S. Moisseiff, C. T. Morris, C. E. Sherman, D. B. Steinman.

WARD, RONALD DAVIES, Jun., Webster Groves, Mo. (Elected Jan. 18, 1926.) (Age 32.) Engr., Eng. Dept., Shell Petroleum Corporation, St. Louis, Mo. Refers to J. B. Butler, D. G. Coombs, E. Flad, G. R. Scott, S. M. Smith.

The Board of Direction will consider the applications in this list not less than thirty days after the date of issue.